A Model of the Functional State of Participants of Laboratory Markets

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Abstract—The dynamics of psycho–physiological characteristics of participants of laboratory markets in the process of making economical decisions in the decentralized control system is analyzed. For this purpose, the method based on the comparison of stabilographic data with the history of market actions recorded in the course of the experiment is used. The key tool for data analysis is the new segmentation algorithm, which provides efficient partitioning of the stabilographic time series into homogeneous fragments. The segmentation algorithm is obtained as the solution of the problem of estimation of the parameters of the hidden Markov model. The application of this algorithm on the level of individual decision making proves the hypothesis of connection of the stabilogram segmentation time instants of the participant with the time instants of signal actions on the laboratory market. On the level of group decisions, the effect of synchronization of stabilographic time series of participants at the time instant of auction culmination connected with revelation of private information is supported. The degree of synchronization is estimated using a proximity factor calculated based on the specially aggregated canonical correlation.

INTRODUCTION

The problem of segmentation of stabilographic time series occurred upon processing of results of biomechanical measurements which were obtained using the system of special chairs developed at special design office "RITM" by the order of the laboratory of experimental economics of the Moscow Institute of Physics and Technology and Dorodnicyn Computing Center, Russian Academy of Sciences. Visually each stabilo-chair does not differ from a common office chair, which allows participants of the experiment to comfortably control the process of making orders at laboratory markets using computers connected into the network. Data from stabilo-chairs are supplied to the special server from which the whole measurement process can be monitored. Highly sensitive sensors installed on each chair fix the change of position of the center of gravity of the participant body with a frequency of 50 times per second. Psycho-physiologists consider this information an important system characteristic of the functional state of the participant [1].

The fragment of the stabilographic time series obtained for one of the participants of the experiment is shown in Fig. 1. Conventionally X corresponds to the left–right motion of the body, Y, to forward–backward, and Z, to up–down which is related with the sitting person leaning on his legs. It can be seen from Fig. 1 that each time series is naturally partitioned into rather homogeneous segments. Inside each segment oscillations about an average value take place. Then the jump is observed and the signal is stabilized on a new level. Jumps which separate one fragment from another one are nonuniform.

This structure of the time series does not allow efficient application of partitioning into equal segments (for example, 30 s each) and the sliding window for calculation of aggregated factors. This yields the problem of construction of an efficient numerical algorithm providing automatic separation of sufficiently homogeneous fragments convenient for subsequent processing. This algorithm was created based on the hidden Markov model.

This approach to segmentation was generalized, as applied to the group stabilography. It was important to develop a tool for estimation of the mutual dependence of functional states of participants in the course of making economic decisions on the laboratory market.

Algorithms of segmentation filtering for several time series and calculation of the factor based on the specially aggregated canonical correlation coefficients, which estimate the likelihood of functional states of participants for segments in the case of group stabilography, were obtained.

The developed algorithm is applied to analysis of the dynamics of the functional state of participants in the course of one particular experiment on the information efficiency of markets performed at the laboratory of experimental economics. The segmentation method was used to achieve the proof of the hypothesis on the connection of segmentation time instants of the stabilogram of a participant with the time instants of

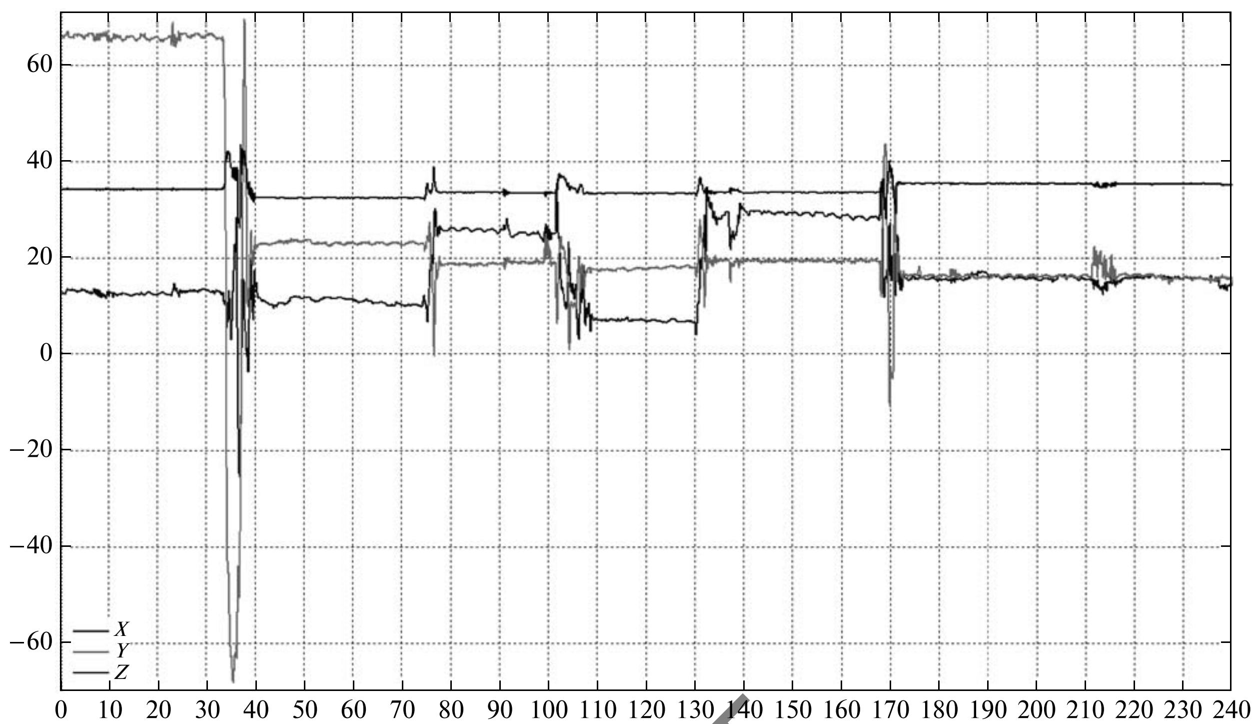

Fig. 1. Time scan of X, Y, and Z coordinates.

signal actions on the laboratory market. Signal actions on the market with private information correspond to the idea of signal strategies in dynamic games with incomplete information [2, 3].

It was possible to analyze the stabilographic time series of all participants of the laboratory market. This made it possible to prove the hypothesis of synchronization of functional states of participants of the laboratory market at the time instant of natural culmination; in the experiment on information efficiency this culmination is connected with the time instant of revelation of private information and prices reaching the equilibrium level with rational expectations [1, 4].

1. TIME SERIES SEGMENTATION ALGORITHM

The following time series segmentation problem is solved: a given time series should be partitioned into several adjacent segments such that data inside each of these segments are homogeneous, but data in adjacent segments, inhomogeneous (homogeneity and inhomogeneity are established using the predetermined criterion).

Segmentation problems occur in many fields of science and technology. The main methods for their solution can be divided into two categories [5]: sequential (on-line methods) and posterior (off-line methods). In this paper, the a posteriori approach to the time series segmentation is developed.

In most posterior methods either the case of two segments is considered [5], or iterative numerical algorithms are used in which the time of execution of one iteration has the order $O\left(N^2\right)$, where N is the length of the known realization of the time series [6]. It was necessary to develop a posterior method of the time series segmentation which, on the one hand, provided the partitioning of the time series into an arbitrary number of segments, and on the other hand, had lower time complexity.

In the proposed method based on the hidden Markov model [7] the partitioning is obtained due to the maximization of the likelihood function of the hidden Markov model. This partitioning is "optimal", since from the probabilistic point of view it corresponds to the local maximum of the likelihood function, and from the numerical point of view this partitioning minimizes the sum of squared deviations of the values of the time series from its average values calculated for the corresponding segments of homogeneity. Note that the application of the hidden Markov model does not mean the simulation of the time series using this model. The hidden Markov model is applied as a tool for obtaining a numerically efficient algorithm for the time series segmentation.

1.1. Segmentation Based on Maximization of Likelihood Function of Hidden Markov Model

Let us reformulate the problem of time series segmentation in terms of the problem of maximization of the likelihood function of some hidden Markov model.

Let $\mathbf{X}^N = (x_1,\dots,x_N)$ be a known realization of the time series. We say that $\mathbf{X}^N$ is a realization of the observed part of $X = (X_i)_{i=1,2,\dots}$ of a hidden Markov model $(S,X) = (S_i,X_i)_{i=1,2,\dots}$ if the process $S = (S_i)_{i=1,2,\dots}$ is the homogeneous Markov chain with M states, i.e., $S_i \in \{1,2,\dots,M\}$. In this case, it is assumed that it is most probable that the initial state of the Markov chain $S_0 = 1$. Let $P_M = (p_{i,j}) \in R^{M\times M}$ be the matrix of transition probabilities of the Markov chain for which $p_{i,j} = 0$ for $|i-j| \ge 2$, $p_{M,M} = 1$. Obviously,

$$\mathrm{P}(S_1 = s_1,\dots,S_N = s_N) = \prod_{i=1}^{N} p_{s_{i-1},s_i} \quad (s_0 = 1).$$

The parameters of the Markov chain S are M and P_M.

The process X is a sequence of conditionally independent (for fixed values of the process S) distributed as $N(\mu_S,\sigma^2)$ random variables, i.e.,

$$\mathrm{P}(X_1 \le x_1,\dots,X_n \le x_n \mid S_1 = s_1,\dots,S_n = s_n)$$
$$= (1/(\sqrt{2\pi\sigma^2})^n \int_{-\infty}^{x_1}\dots\int_{-\infty}^{x_n}\prod_{i=1}^{n}\exp[-(z_i-\mu_{s_i})^2/2\sigma^2]$$
$$\times dz_1\dots dz_n.$$

Thus, the parameters of the process $(S,X) = (S_i,X_i)_{i=1,2,\dots}$ are the vector of average values $\mu = (\mu_1,\dots,\mu_M)$, the variance σ, and the transition probability matrix P_M.

Let $J(\mathbf{z})$ be the function equal to the number of components with different values in the arbitrary vector $\mathbf{z} \in R^N$. Let us denote by $\mathbf{s}_N = (s_1,\dots,s_N)$ the unobserved realization of the values of Markov chain corresponding to the realization of the observed part $\mathbf{X}^N = (x_1,\dots,x_N)$ of the hidden Markov model. We assume that $M_s = J(\mathbf{s}_N)$.

The vector $\mathbf{n}_s = (n_0,n_1,\dots,n_{M_s})$ $(n_0 = 0, n_{M_s} = N)$ of coordinates of the homogeneity segments of the time series is a vector for whose coordinates $n_i, i = 1,\dots,M_s - 1$, the following inequality is satisfied: $s_{n_i} \ne s_{n_i+1}$. Obviously, $M_s \le M$ and each inhomogeneity segment $[n_{i-1},n_i]$ corresponds to the state i of the Markov chain S and the parameter μ_i of the distribution of values of the process X. Thus, estimating the realization $\mathbf{s}_N = (s_1,\dots,s_N)$ of the process S using the realization $\mathbf{X}^N$ of the process X, it is possible to obtain an estimate of the vector $\mathbf{n}_s = (n_0,n_1,\dots,n_{M_s})$ and thus segment the signal into homogeneous (in the sense of constant average value) fragments.

The realization $\mathbf{s}_N$ will be estimated using the maximization of likelihood of $\mathbf{s}_N$ for fixed $\mathbf{X}^N$. Denoting the conditional likelihood function as $L(\mathbf{s}_N \mid \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$, we find that

$$\mathbf{s}_N^{opt} = \arg\max_{\mathbf{s}_N\in\{1,2,\dots,M\}^N} L(\mathbf{s}_N \mid \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma).$$

The joint likelihood function $L(\mathbf{s}_N, \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$ of the quantities $\mathbf{s}_N$ and $\mathbf{X}^N$ is connected with the conditional likelihood function $L(\mathbf{s}_N \mid \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$ according to the Bayes formula

$$L(\mathbf{s}_N \mid \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$$
$$= L(\mathbf{s}_N, \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma) / F(\mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma),$$

where $F(\mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$ is the unconditional density of the distribution $\mathbf{X}^N$. Therefore, the maximization of $L(\mathbf{s}_N \mid \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$ with respect to $\mathbf{s}_N$ is equivalent to the maximization of $L(\mathbf{s}_N, \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$ with respect to $\mathbf{s}_N$, where

$$L(\mathbf{s}_N, \mathbf{X}^N; M, P_M, \boldsymbol{\mu}, \sigma)$$
$$= \left(1/(\sqrt{2\pi}\sigma)^N\right)\prod_{i=1}^{N}\left(p_{s_{i-1},s_i}\exp\left(-(x_i-\mu_{s_i})^2/2\sigma^2\right)\right) \tag{1.1}$$

Thus, the segmentation problem is reduced to the problem of maximization of function (1.1).

1.2. Description of Segmentation Algorithm

Let us assume that for elements of the matrix P_M the following equalities are satisfied: $p_{i,i} = p$ and $p_{i,i+1} = 1 - p$ for $i = 1,\dots,M-1$, $p_{M,M} = 1$ for a parameter $p \in (0,1)$. In what follows, the parameter p will be indicated in the arguments of the functions instead of the matrix P_M.

The input data for the segmentation algorithm are:

(1) $\mathbf{X}^N = (x_1,\dots,x_N)$ is a known realization of the time series;

(2) upper estimate of the number of segments M $(M \le N/2)$;

(3) threshold value of ε.

The output data are the estimates of the parameters p^{opt}, $\boldsymbol{\mu}^{opt}$, σ^{opt}, $\mathbf{s}_N^{opt}$, $\mathbf{n}_s^{opt}$, and $M_s^{opt} = J(\mathbf{s}_N^{opt})$. In this case, generally speaking, it may turn out that $M_s^{opt} < M$.

Initialization of the algorithm:

(1) $p^{(0)} = (N-M)/N$;

(2) $\mathbf{s}_N^{(0)}$ is randomly generated from elements of the set $\{1,2,\ldots,M\}$ in such a way that $J\left(\mathbf{s}_N^{(0)}\right) = M$ and the components of the vector $\mathbf{s}_N^{(0)}$ satisfy the inequalities $s_1 \le s_2 \le \ldots \le s_N$,

$$\sigma^{opt} = \sqrt{(1/(N-1))\sum_{i=1}^{N}(x_i - \bar{x})^2},$$

where $\bar{x} = (1/N)\sum_{i=1}^{N} x_i$.

Condition of algorithm termination. Let I be the current iteration number of the algorithm. The algorithm is terminated if

$$\left| \begin{array}{l} L\left(\mathbf{s}_N^{(I)}, \mathbf{X}^N; M_s^{(I)}, p^{(I)}, \boldsymbol{\mu}^{(I)}, \sigma^{opt}\right) \\ -L\left(\mathbf{s}_N^{(I-1)}, \mathbf{X}^N; M_s^{(I-1)}, p^{(I-1)}, \boldsymbol{\mu}^{(I-1)}, \sigma^{opt}\right) \end{array} \right| < \varepsilon.$$

At each iteration I of the algorithm the following steps are **executed**.

Step 1. $\mathbf{s}_N^{(I)}$ is used to determine the vector $\mathbf{n}_s^{(I)}$ and $M_s^{(I)} = J\left(\mathbf{s}_N^{(I)}\right)$.

Step 2. $\mathbf{n}_s^{(I)}$ is used to calculate $\mu^{(I)} \in R^{M_s^{(I)}}$, where

$$\mu_i = (1/(n_i - n_{i-1}))\sum_{k=n_{i-1}+1}^{n_i} x_k, \quad i = 1,\ldots, M_s^{(I)}. \tag{1.2}$$

Step 3. The probability $p^{(I)} = \left(N - M_s^{(I)}\right)/N$ is calculated.

Step 4. The value of the likelihood function $L\left(\mathbf{s}_N^{(I)}, \mathbf{X}^N; M_s^{(I)}, p^{(I)}, \boldsymbol{\mu}^{(I)}, \sigma^{opt}\right)$ is estimated and the **condition of the algorithm termination** is checked. If the termination condition is true, $p^{opt} = p^{(I)}$, $\boldsymbol{\mu}^{opt} = \boldsymbol{\mu}^{(I)}$, $\mathbf{s}_N^{opt} = \mathbf{s}_N^{(I)}$, $\mathbf{n}_s^{opt} = \mathbf{n}_s^{(I)}$, and $M_s^{opt} = M_s^{(I)} = J\left(\mathbf{s}_N^{(I)}\right)$, otherwise, the transition to Step 5 is performed.

Step 5. The following quantity is calculated:

$$\mathbf{s}_N^{(I+1)} = \arg \max_{\mathbf{s}_N \in \left\{1,2,\ldots,M_s^{(I)}\right\}^N} L\left(\mathbf{s}_N, \mathbf{X}^N; M_s^{(I)}, p^{(I)}, \boldsymbol{\mu}^{(I)}, \sigma^{opt}\right)$$

using the Viterbi algorithm [7, 8]. The iteration number increases by unity, i.e., $I \to I+1$, and the process is repeated from Step 1.

The proposed algorithm in essence belongs to the class of the so-called EM algorithms [9]. Note that in the case of numerical implementation of the algorithm it is reasonable to consider the logarithm of the initial value of the likelihood function, rather than the value of the function.

1.3. Convergence of Segmentation Algorithm

Let us prove that the proposed algorithm converges. From (1.1) we find that

$$F\left(\mathbf{s}_N, \mathbf{X}^N; M, p, \boldsymbol{\mu}, \sigma\right) = -\ln L\left(\mathbf{s}_N, \mathbf{X}^N; M, p, \boldsymbol{\mu}, \sigma\right)$$
$$= -\sum_{i=1}^{N} \ln\left(p_{s_{i-1},s_i}\right) + \sum_{i=1}^{N}\left(x_i - \boldsymbol{\mu}_{s_i}\right)^2/2\sigma^2 + N\ln\left(\sqrt{2\pi}\sigma\right),$$

$$F\left(\mathbf{s}_N, \mathbf{X}^N; M, p, \boldsymbol{\mu}, \sigma\right) = \frac{1}{2\sigma^2}\left[F_1\left(\mathbf{s}_N, \mathbf{X}^N; M, \boldsymbol{\mu}\right) + 2\sigma^2 F_2(\mathbf{s}_N; M, p)\right] + N\ln\left(\sqrt{2\pi}\sigma\right), \tag{1.3}$$

where $F_1\left(\mathbf{s}_N, \mathbf{X}^N; M, \mu\right) = \sum_{i=1}^{N}\left(x_i - \mu_{s_i}\right)^2$, $F_2(\mathbf{s}_N; M, p) = J(\mathbf{s}_N)\ln(p/(1-p)) - N\ln p$.

Obviously for all $\boldsymbol{\mu} \in R^{M_s^{(I)}}$, where $M_s^{(I)} = J\left(\mathbf{s}_N^{(I)}\right)$ the following is satisfied: $F_1\left(\mathbf{s}_N^{(I)}, \mathbf{X}^N; M_s^{(I)}, \boldsymbol{\mu}\right) \ge F_1\left(\mathbf{s}_N^{(I)}, \mathbf{X}^N; M_s^{(I)}, \boldsymbol{\mu}^{(I)}\right)$, where the components $\boldsymbol{\mu}^{(I)}$ are calculated using formula (1.2). Similarly for all $p \in (0,1)$ the following relation is satisfied: $F_2\left(\mathbf{s}_N^{(I)}; M_s^{(I)}, p\right) \ge F_2\left(\mathbf{s}_N^{(I)}; M_s^{(I)}, p^{(I)}\right)$, where $p^{(I)} = \left(N - M_s^{(I)}\right)/N$. Since the Viterbi algorithm provides the global maximum, $F\left(\mathbf{s}_N, \mathbf{X}^N; M_s^{(I-1)}, p, \boldsymbol{\mu}^{(I-1)}, \sigma\right) \ge F\left(\mathbf{s}_N^{(I)}, \mathbf{X}^N; M_s^{(I-1)}, p, \boldsymbol{\mu}^{(I-1)}, \sigma\right)$ for all $\mathbf{s}_N \in \left\{1,2,\ldots,M_s^{(I-1)}\right\}^N$. Thus, we find that

$$F\left(\mathbf{s}_N^{(I-1)}, \mathbf{X}^N; M_s^{(I-1)}, p^{(I-1)}, \boldsymbol{\mu}^{(I-1)}, \sigma\right)$$
$$\ge F\left(\mathbf{s}_N^{(I)}, \mathbf{X}^N; M_s^{(I-1)}, p^{(I-1)}, \boldsymbol{\mu}^{(I-1)}, \sigma\right)$$
$$\ge F\left(\mathbf{s}_N^{(I)}, \mathbf{X}^N; M_s^{(I)}, p^{(I)}, \boldsymbol{\mu}^{(I)}, \sigma\right)$$
$$\ge F\left(\mathbf{s}_N^{(I+1)}, \mathbf{X}^N; M_s^{(I)}, p^{(I)}, \boldsymbol{\mu}^{(I)}, \sigma\right).$$

Therefore, each iteration of the proposed algorithm decreases the function $F\left(\mathbf{s}_N, \mathbf{X}^N; M, p, \boldsymbol{\mu}, \sigma\right)$ which is bounded from below by zero, i.e., the iteration process converges.

1.4. Properties of Segmentation Algorithm

Let us indicate the main properties of the proposed segmentation algorithm.

P r o p e r t y 1. The time of execution of one iteration of the algorithm is $O\left(N \cdot M^2\right)$ [10]. In numerical experiments the algorithm converged faster than for 10–15 iterations, which is typical for algorithms of EM type [10].

P r o p e r t y 2. The function $F_2(\mathbf{s}_N; M, p)$ takes the minimal value equal to $F_2\left(\mathbf{s}_N; M, p^{opt}\right)$ = $J(\mathbf{s}_N)\ln(N/J(\mathbf{s}_N)-1) - N\ln(1-J(\mathbf{s}_N)/N)$ for p^{opt} = $(N - J(\mathbf{s}_N))/N$. If $J(\mathbf{s}_N) \le M \ll N$, then $2\sigma^2 F_2\left(\mathbf{s}_N; M, p^{opt}\right) \approx 2\sigma^2 J(\mathbf{s}_N)(\ln N + O(M/N) + O(\ln M))$ and it asymptotically (for $M \ll N$) coincides with the second term in the Schwarz criterion, which is applied in statistics for estimation of the number of parameters of a model [9]. In this case the term $2\sigma^2 F_2\left(\mathbf{s}_N; M, p^{opt}\right)$ in (1.3) is responsible for regularization, since it increases with increasing $J(\mathbf{s}_N)$ (for $J(\mathbf{s}_N) \le M \le N/2$), while the term $F_1\left(\mathbf{s}_N, \mathbf{X}^N; M, \boldsymbol{\mu}\right)$ decreases (and vice versa). Therefore, if the Viterbi algorithm is used for minimization of $F\left(\mathbf{s}_N, \mathbf{X}^N; M, p, \boldsymbol{\mu}, \sigma\right)$ with respect to $\mathbf{s}_N \in \{1, 2, \ldots, M\}^N$ for fixed values of M and $\boldsymbol{\mu}$ the optimal number of segments equal to $M_s = J(\mathbf{s}_N) \le M$ is also chosen. Generally speaking, $M \ge J\left(\mathbf{s}_N^{(0)}\right) \ge J\left(\mathbf{s}_N^{(1)}\right) \ge J\left(\mathbf{s}_N^{(2)}\right) \ge \ldots$, and the process of decreasing the value of $J\left(\mathbf{s}_N^{(I)}\right)$ beginning from some iteration I stabilizes, since otherwise the value of $F\left(\mathbf{s}_N, \mathbf{X}^N; M, p, \boldsymbol{\mu}, \sigma\right)$ begins to increase.

P r o p e r t y 3. As soon as the value of $M_s^{(I)} = J\left(\mathbf{s}_N^{(I)}\right)$ stabilizes, $F_1\left(\mathbf{s}_N, \mathbf{X}^N; M_s^{(I)}, \boldsymbol{\mu}\right)$ decreases due to the "tuning" of the partitioning $\mathbf{n}_s$ (and therefore, "tuning" of the values of $\boldsymbol{\mu}$). Therefore, the proposed algorithm, in essence, seeks such partitioning for which the approximation of the time series by its average values for the corresponding segments is optimal in the sense of the mean square deviation.

P r o p e r t y 4. Numerical experiments have demonstrated that the results of operation of the algorithm are weakly dependent on the initial values of $p^{(0)}$ and $\mathbf{s}_N^{(0)}$, but they depend on M. It turned out that if $M \ll N$, with high probability $M_s^{opt} = J\left(\mathbf{s}_N^{opt}\right) = M$, otherwise $M_s^{opt} < M$, and for some value of $M = M_1$ the optimum corresponds to $M_s^{opt,1}$, and the other value of $M = M_2 > M_1$ corresponds to $M_s^{opt,2}$ which, generally speaking, is not equal to $M_s^{opt,1}$. This is related with the fact that the minimized function is multi-extremal.

1.5. Segmentation of a Multidimensional Time Series

The proposed segmentation algorithm can easily be generalized to the case of the multidimensional time series. Let $\mathbf{X}^N = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ be a known realization of the d-dimensional time series with the length N, where $\mathbf{x}_i = \left(x_i^1, \ldots, x_i^d\right)$ is the value of the d-dimensional component at the ith time instant. It will be assumed that $\mathbf{X}^N$ is a realization of the observed part of $\mathbf{X} = (\mathbf{X}_i)_{i=1,2,\ldots}$ of the hidden Markov model $(S, \mathbf{X}) = (S_i, \mathbf{X}_i)_{i=1,2,\ldots}$, where the process $S = (S_i)_{i=1,2,\ldots}$ is the homogeneous Markov chain with M states determined in Sections 1.1 and 1.2.

Let us also assume that for the fixed realization $\mathbf{s}_N = (s_1, \ldots, s_N)$ of the Markov chain the values of the process $\mathbf{X}$ are independent; in this case, the distribution of the value of the process $\mathbf{X}$ at the ith time instant is the multidimensional normal $N(\boldsymbol{\mu}_{s_i}, \Sigma)$, $i = 1, \ldots, N$, where $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1, \ldots, \sigma_d)$ and $\boldsymbol{\mu} \in \{\boldsymbol{\mu}_1, \ldots, \boldsymbol{\mu}_M\}$, $\boldsymbol{\mu}_i = \left(\mu_i^1, \ldots, \mu_i^d\right)$, $i = 1, \ldots, M$ (the set of all possible values of the mathematical expectation $\boldsymbol{\mu}$). For these assumptions, the segmentation of the multidimensional time series can be performed using an algorithm similar to the algorithm of segmentation of one-dimensional time series described in Section 1.2. In this case the following function is used as the likelihood function:

$$L\left(\mathbf{s}_N, \mathbf{X}^N; M, p, \mu, \sigma\right) = \left(\prod_{j=1}^{d} 1/\left(\sqrt{2\pi}\sigma_j\right)^N\right) \times \prod_{i=1}^{N}\left(p_{s_{i-1}, s_i} \prod_{j=1}^{d} \exp(-(x_i^j - \mu_{s_i}^j)^2/2\sigma_j^2)\right).$$

2. ESTIMATE OF DEGREE OF SYNCHRONIZATION OF COMPONENTS OF MULTIDIMENSIONAL PROCESS FOR FILTERED SEGMENTATION

Let us solve the problem of estimation of the degree of synchronization of components of the multidimensional time series. The degree of synchronization is estimated in three steps.

S t e p 1. Using the segmentation algorithm described in Section 2.1, we estimate using the realization $\mathbf{X}^N$ of the process $\mathbf{X}$ the realization $\mathbf{s}_N = (s_1, \ldots, s_N)$

of the Markov chain S and obtain an estimate of the vector $\mathbf{n}_s = (n_0, n_1, \ldots, n_{M_s})$ of coordinates of segments.

Step 2. We filter the obtained segmentation $\mathbf{n}_s = (n_0, n_1, \ldots, n_{M_s})$ (see Section 2.2 for description of the algorithm) in order to save only those segments that correspond to the time instants of the simultaneous significant changes of the average values of all components of the process $\mathbf{X}$.

Step 3. We calculate the synchronization coefficients (based on the canonical correlation coefficient) of the multidimensional process for filtered segmentation (see Section 1.2 for description of the algorithm).

2.1. Segmentation Filtering for a Multidimensional Process

It is obvious that in the vector of coordinates of segments $\mathbf{n}_s = (n_0, n_1, \ldots, n_{M_s})$ the coordinate of the ith segment n_i corresponds to such time instant that the average value of at least one of the components of the process $\mathbf{X}$ at the n_ith time instant changes by a sufficiently large magnitude. In this case it may turn out that the average values of other components are unchanged. Therefore, in order to estimate the degree of synchronization of components of the multidimensional process $\mathbf{X}$, it is necessary, first of all, to "filter" the obtained segmentation $\mathbf{n}_s = (n_0, n_1, \ldots, n_{M_s})$ of the multidimensional process $\mathbf{X}$, i.e., to include into the filtered segmentation only the time instants n_i that correspond to the time instants of simultaneous significant change of average values of all components of the process $\mathbf{X}$.

The input data for the filtering algorithm are:

$\mathbf{X}^N = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ is a known realization of the time series,

$\alpha > 0$ and $1 \le \delta \le \min\limits_{i=1,\ldots,M_s} (n_i - n_{i-1})$ are the parameters of the filtering algorithm.

The output data of the segmentation filtering are

$\tilde{\mathbf{n}}_s = (\tilde{n}_0, \tilde{n}_1, \ldots, \tilde{n}_{\tilde{M}_s})$, where $\tilde{M}_s \le M_s$, $\tilde{n}_0 = n_0 = 1$, $\tilde{n}_{\tilde{M}_s} = n_{M_s} = N$.

Initialization of the algorithm: the following values are estimated: $\sigma^j = (1/(N-1)) \sum\limits_{i=1}^{N} \left(x_i^j - \bar{x}^j\right)^2$, where $\bar{x}^j = (1/N) \sum\limits_{i=1}^{N} x_i^j$, $j = 1, \ldots, d$.

During the Ith **iteration** of the algorithm ($I = 1, \ldots, M_s - 1$) the following steps **are performed**.

Step 1. The following is calculated: $\Delta_j^I = \max\limits_{i,k \in [n_I - \delta, n_I + \delta]} \left|x_i^j - x_k^j\right|$, $j = 1, \ldots, d$.

Step 2. If $\Delta_j^I \ge \alpha \sigma_j$ for all $j = 1, \ldots, d$, the coordinate of the segment n_I is included into the filtered partitioning $\tilde{\mathbf{n}}_s$.

2.2. Estimate of Synchronization Based on Aggregated Canonical Correlation

Let $\tilde{\mathbf{n}}_s = (\tilde{n}_0, \tilde{n}_1, \ldots, \tilde{n}_{\tilde{M}_s})$ be the filtered partitioning of the realization $\mathbf{X}^N = (\mathbf{x}_1, \ldots, \mathbf{x}_N)$ of the d-dimensional time series. It is necessary to estimate the degree of synchronization of variation of the values of components of the process $\mathbf{X}$ on the time intervals $[\tilde{n}_{i-1}, \tilde{n}_i]$, $i = 1, \ldots, \tilde{M}_s$. We use the canonical correlation coefficient [11] for this purpose.

First of all, let us give the definition of the canonical correlation coefficient. Let $\mathbf{Y} \in R^q$ and $\mathbf{Z} \in R^q$ be two multidimensional random variables, and $\alpha \in R^p$ and $\gamma \in R^q$ are arbitrary vectors. Then the canonical correlation coefficient is

$$\rho = \max_{\alpha, \gamma : \mathbf{Var}((\alpha, \mathbf{Y}))=1, \mathbf{Var}((\gamma, \mathbf{Z}))=1} \mathbf{corr}((\alpha, \mathbf{Y}), (\gamma, \mathbf{Z})), \qquad (2.1)$$

where $\mathbf{corr}(\cdot,\cdot)$ is the correlation operator, $\mathbf{Var}(\cdot)$ is the variance operator, and $(\alpha, \mathbf{Y})$ and $(\gamma, \mathbf{Z})$ are the scalar products.

We denote by ρ_i^j a chosen canonical correlation coefficient (in (2.1) the variance and correlation operators are replaced by the corresponding chosen analogues) between the jth and all other components of the process $\mathbf{X}$ calculated for the part of the realization $(\mathbf{x}_{\tilde{n}_{i-1}}, \mathbf{x}_{\tilde{n}_{i-1}+1}, \ldots, \mathbf{x}_{\tilde{n}_i})$, $i = 1, \ldots, \tilde{M}_s$. We estimate the degree of synchronization of variation of the values of components of the process $\mathbf{X}$ on the time intervals $[\tilde{n}_{i-1}, \tilde{n}_i]$, $i = 1, \ldots, \tilde{M}_s$ using the synchronization coefficients

$$\rho_i^{L_2} = \sqrt{\sum_{j=1}^{d} \left(\rho_i^j\right)^2 / d} \text{ and } \rho_i^{L_\infty} = \max_{j=1,\ldots,d} \left|\rho_i^j\right|.$$

Obviously, if on some time interval $[\tilde{n}_{i-1}, \tilde{n}_i]$, $i = 1, \ldots, \tilde{M}_s$, the coefficients $\rho_i^{L_2}$ and $\rho_i^{L_\infty}$ take values close to 1 and all values of the correlation coefficients ρ_i^j, $j = 1, \ldots, d$, are significant, then on this time interval considerable synchronization of components of the process $\mathbf{X}$ is observed.

3. INDIVIDUAL FUNCTIONAL STATE IN DECISION MAKING

On laboratory markets, the decision making process consists of the sequence of trading actions of participants. Each action corresponds to the order which should be input at a computer using the mouse and the keyboard, and then via the network sent to the server that supports trading for a certain variety of the auction. In this paper, we consider an open continuous double auction which is the basis of operation of most stock exchanges.[1] The main trading actions of the participant are the request for buying or selling for the given trading position (type of security or commodity). Let us consider the simplest experiment RE0 of the series of information efficiency of markets in which there is the only trading position, namely, some venture asset. It is known to all participants that at the end of the auction the value of this asset will be 30, 70, and 110 with equal probability. It can be assumed for simplicity that the participants buy and sell to each other lottery tickets whose drawing will take place at the end of the auction.

The specific feature of experiments of RE (Rational Expectations) series is that along with the common information any participant receives reliable private information. In RE0 each participant knows which of the three scenarios cannot take place in a particular case. For example, if the organizers know that the scenario with a value of 70 is realized, some participants are told that the asset will not cost 30, and other participants are told that it will not cost 110. See [4] for detailed analysis of experiment of RE series.

For establishing the connection of the records of trading actions of the participants fixed in the course of the experiment each second with the chronology of the functional state expressed by the stabilogram with a frequency of 50 times per second, it is necessary to introduce the significance of the action for the participant. For RE0 experiment this is rather simple, since there are three separate price levels. If the price of the request for buying is smaller than 30, it can be considered as the desire of the participant to play using someone's evident mistake or just as a beginning of the auction. As the request for buying becomes higher than 30, this is the signal to other participants. What does it mean? Three variants of answers are possible: (1) the author of the request possesses information that a value of 30 will not be realized; (2) this author made a mistake or did not think well enough; (3) this author is sly and tries to confuse others. The answer to this question is sought by everyone individually.

Thus, in RE0 the signal actions are assumed to be the first (with respect to time) requests for buying at a price higher than 30, 70, and 110 (the last request for buying at a price higher than 110 is the evident mistake). The signal actions are also the sharp change of the request price (larger than by some threshold value). Our hypothesis is that the special reflection in the stabilogram of the participant should be sought in the neighborhood of those time instants when the participant performs the signal action or observes the manifestation of the signal action of some other participant.

Let us consider the RE0 experiment performed at the Laboratory of Experimental Economics of the Moscow Institute of Physics and Technology on December 12, 2006. Figure 2 shows the prices of all buying and selling requests. Time in seconds is shown along the horizontal axis. The true value of the asset which was told to the participants after the end of the auction that lasted for 240 s in this case was 30.

At the 21st second, the first signal action took place: the request for selling 10 items at a price of 109.9 appeared. This request was made by participant 5 who possessed the information against a value of 110. This signal was true, but not everyone believed it at once.

At the 24th second, another signal action took place: the request for buying 1 item at a price of 30.1 Participant 2 who performed this signal action possessed the information that a value of 70 was impossible; therefore, his actions can be considered as the reconnaissance. If anyone would sell 1 item at a price of 30.1 to him, his loss at the end of the auction would make just 0.1.

The next signal action was observed at the 47th second, when participant 2 sharply increased the price of buying from 32.3 to 50. This means that either participant 2 came to believe that a value of 30 will not be realized, or risked (a loss of 20 for each bought item) in order to "carry" someone together with him.

At that time the flow of requests for prices below 110 and then below 105 and 100 increased, and participant 2 made the final choice in favor of the true scenario that assumed a value of 30. He removed all his requests for buying and beginning from the 84th second started to sell at a price of 50 and higher according to requests for buying of other participants.

As a result, at the 45th second the request for buying vanished from the market. Requests for selling smoothly decreased to a price level of 95 until the same participant dropped the selling price to 80 at the 127th second. After another 15 seconds of decrease, participant 4, who possessed information against a value of 70 and at that time did not believe in a value of 110, decreased the price to 70. After another 15 seconds of decrease of the selling price, the trade stopped: no more requests were made. Buying requests appeared, but on a price level below 30.

After 10 seconds of global reflection participant 2 submitted the selling request at a price of 40 at the 174th second, and the price on the market began to fall. It became clear to everybody that the true scenario in this case was a value of 30.

At the end of the experiment participant 5 lost orientation and began to sell actively beginning from the 199th second at prices lower than 30; other participants made use of it.

The culmination of the auction in the narrow sense is the slack period from the 160th to the 173th seconds which ended by the powerful signal action. Special

[1] The functional state of participants of laboratory English and Dutch single-side auctions using the analysis of the heart rhythm was studied in an interesting paper [12].

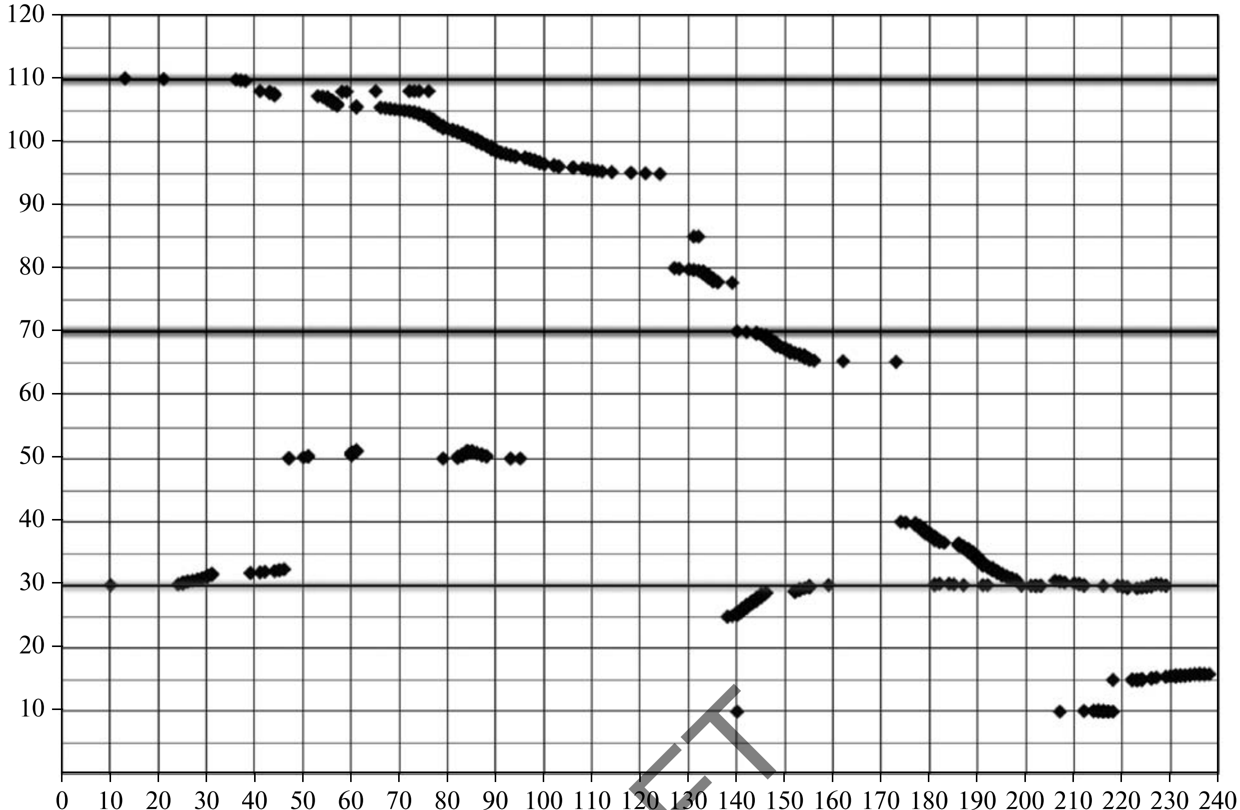

Fig. 2. Request price during the auction.

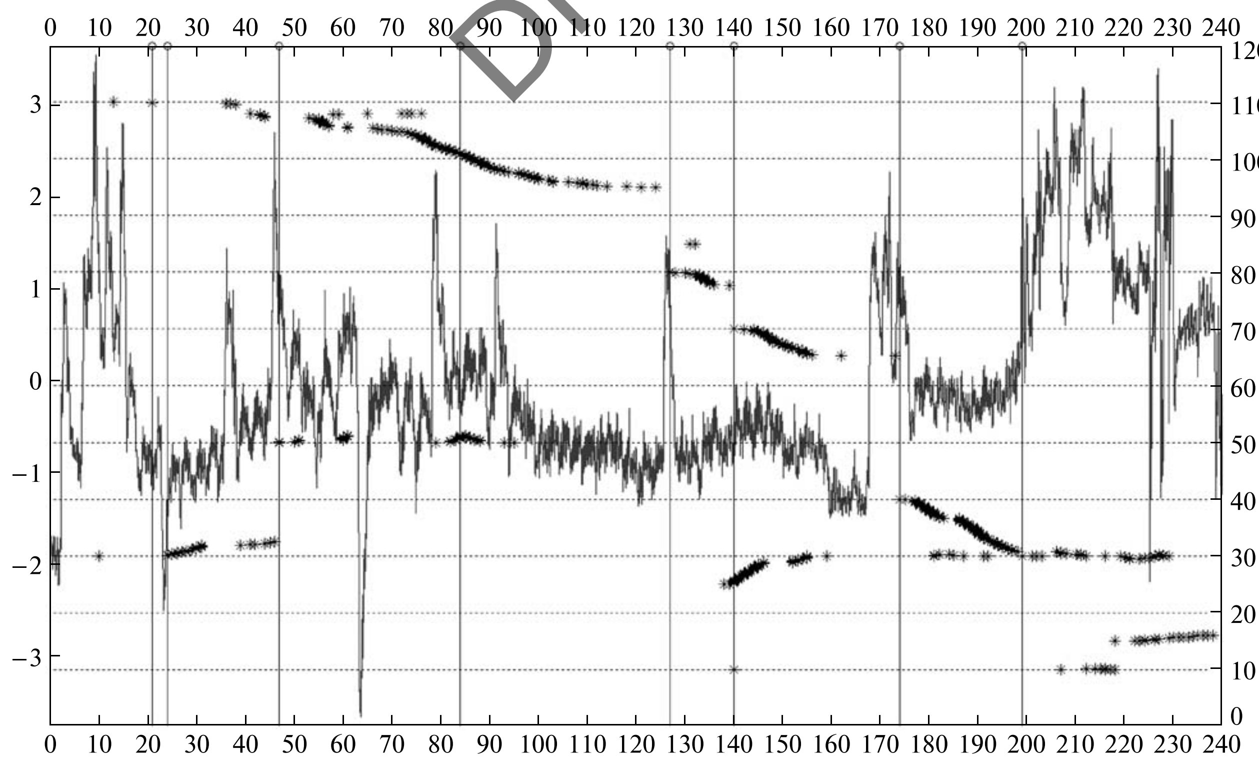

Fig. 3. *Y* coordinate of the stabilogram and signal actions of the participant.

Table 1. Segmentation time instants

Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Time, s	9	36	47	63	84	85	121	123	135	151	171	197	199	208	241

Table 2. Revelation of signal time instants

Number of time instant	1	2	3	4	5	6	7	8
Time, s	21	24	47	84	127	140	174	199
Measure of revelation	12	15	0.18	0.38	3.86	5	3.04	2.26

attention should be paid to the signal action at the 140th second connected with selling at a critical price of 70.

Thus, we separated a total of eight signal actions performed at different time instants $M_A = \{21, 24, 47, 84, 127, 140, 174, 199\}$.

Let us consider the dynamics of the functional state of participants. We begin with participant 2 who played the most active role in this experiment having performed several signal actions.

Figure 3 shows the pronounced jumps along the Ycoordinate of the stabilogram before each signal action. Here, vertical lines corresponding to the found signal actions of the participant are drawn manually based on the detailed analysis of the chronology of trading actions. Note that the last signal near the 200th second corresponded to the first mistake of participant 5 at the final stage of the auction when he began to sell at a price lower than 30. The jump at the first 20 s of the auction is connected with the external signal: the market is opened. It is interesting that the action at the 21st second did not impress participant 2. The jump before the action at the 24th second is hardly visible on the background of the violent initial period when the participant receives private information and is at the stage of formation of the plan of actions.

The task is to perform segmentation of the functional state automatically independently of trading actions, and then correlate the found time instants of segment joining with the time instants of signal trading actions.

In our algorithm, the number of fragments is the external parameter. Figure 3 shows that it is impossible to interpret each jump of the stabilogram by some trading action. Such jumps, although with lower amplitude, are observed when participants of the experiment are asked to sit still (doing nothing) before and after the auction for one half of the minute with opened and one half of the minute with closed eyes.

Since we separated eight signal time instants, this corresponds to nine segments. Let us increase by a factor of 2 the number of segments, assuming that just half of all jumps of the stabilogram are connected with signal actions of the participants.

We use the segmentation algorithm described above for the Y coordinate of the stabilogram of participant 2 (Fig. 4).

All critical time instants mentioned above were successfully separated by the segmentation algorithm directly based on the analysis of the Y coordinate of participant 2, except for those corresponding to time instants of the 21st and 24th seconds. This gives hope that the stabilogram reflects the functional state of the participant of the laboratory market in the process of making decisions.

Let us introduce the formal measure of revelation of the critical time instant $m_A \in M_A$ for the set of segmentation time instants M_S,

$$\rho(m_A, M_S) = \min_{m_S \in M_S} |m_A - m_S|.$$

In our case, the set M_S obtained by the segmentation algorithm is conveniently represented in Table 1.

Table 2 gives the measure of revelation time instants M_A corresponding to signal trading actions of the participants.

It can be seen that the time instants of the 21st and 24th seconds were not found, but all other delays correspond to the psycho–physiologic norm. Note that the segmentation algorithm automatically separated the time instant of the first request (9th second) and the time instant of the end of the auction (241st second). The time instant of the 36th second looks as the reasonable alternative to the time instant of the 21st second. At the 36th, 37th, and 38th seconds participant 5 began to gradually reduce the selling price which supported his single action at the 21st second.

Thus, in this case for participant 2 the segmentation algorithm for the Y coordinate reliably found six out of eight instants (75%). It is important to note that the time instant of the 174th second from the culmination interval was among the revealed signal time instants. Note also that the stabilogram of participant 2 fixed the jump 3 seconds before he performed the decisive action, namely the selling request at a price of 40 at the 174th second. At a time instant of the 171st second (found by the segmentation algorithm) and during 6 s before it nothing took place on the market. It can be assumed that this jump is connected with the cog-

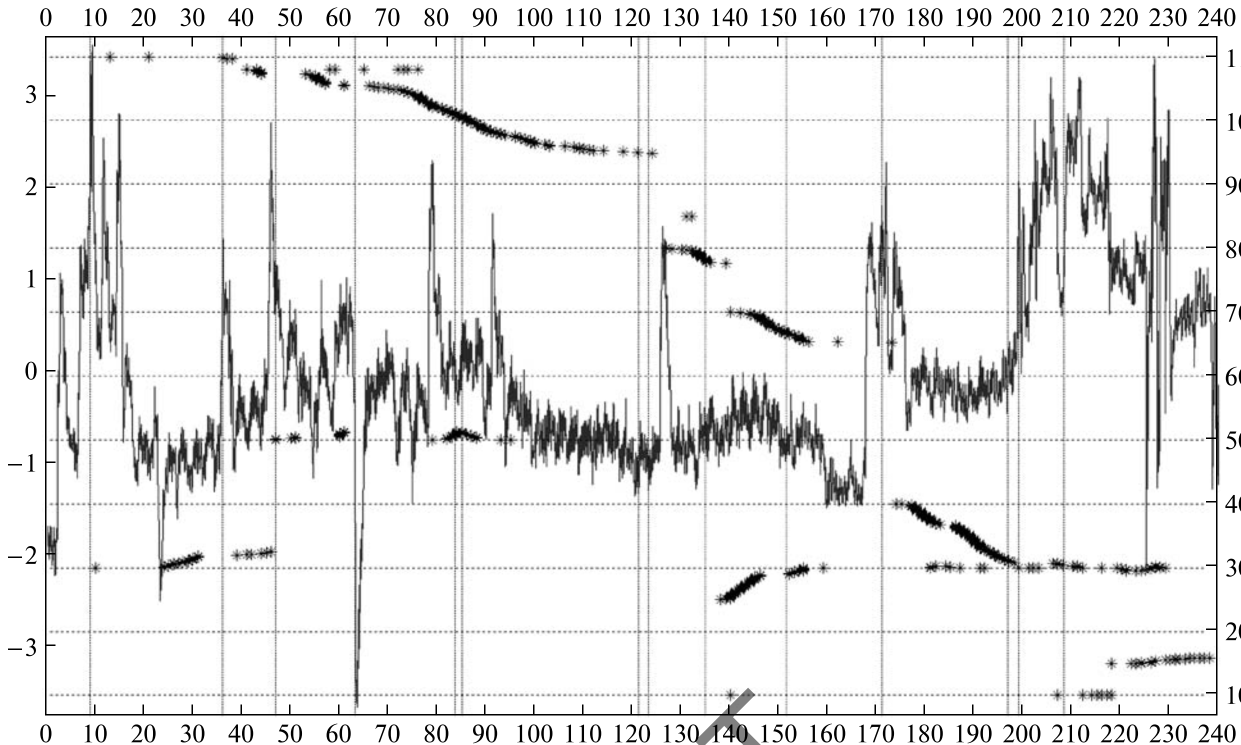

Fig. 4. Request price and automatic segmentation.

nitive load of participant 2 upon making an important decision.

In most cases the Y coordinate (forward–backward movement) is more informative in segmentation than X (left–right) and Z (up–down) coordinates. However, taking into account these coordinates may considerably improve the segmentation. Here, two segmentation methods are possible with account of all three coordinates of the stabilogram: the logical and the physiological ones. The logical method is connected with the introduction of the likelihood function equal to the product of these functions of coordinates. Actually, this approach takes into account significant jumps along any coordinate. The physical method is based on the idea of stabilogram energy [1]: the linear velocity is calculated for two neighboring

Table 3. Degree of revelation of signal time instants using stabilography

Participant		1		2		3		4		5		Revelation, %
Profit		−0.58		0.62		1.16		−1.19		0		
Method		V	F	V	F	V	F	V	F	V	F	
Time instant, s	Signal author											
21	4								−1.06			10
24	2	3.88	4.35			4.68					−0.10	40
47	2	2.68	0.63	−0.08	−1.62	−1.66	−5.16			1.62		70
84	2		−4.58	−1.30	0.94	3.02	3.24	−1.66		−1.44		70
127	2	−3.70	−0.08	−1.48	−1.50	0.32	2.58					60
140	5			−2.16	0.14	−0.34	1.16			−6.54	−4.00	40
174	2	0.75	−9.13	0.74	1.24	3.02	3.50	0.12	0.24	−0.80		90
199	4	−3.22		−0.28	−0.26	7.72	−0.98	−0.78	0.76	0.68	−3.00	90
	Revelation, %	62.5	62.5	75	75	87.5	75	37.5	37.5	62.5	37.5	

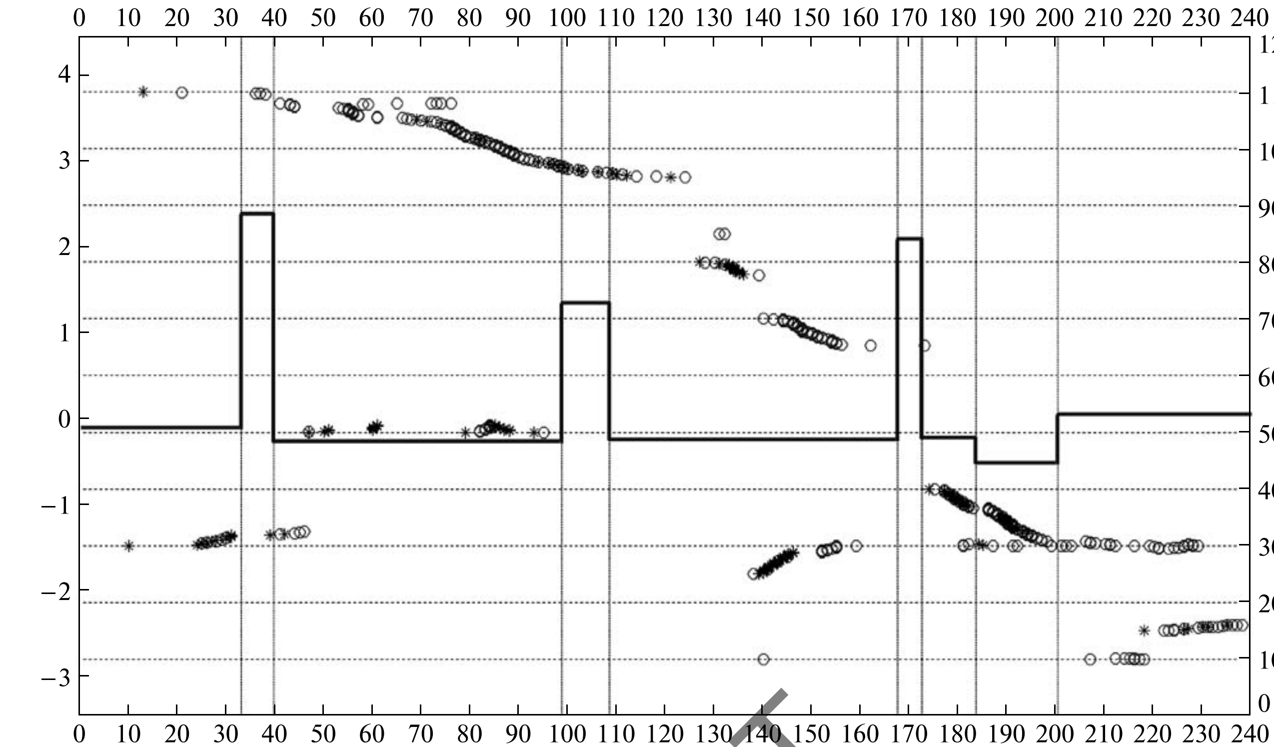

Fig. 5. Total energy of stabilograms and request price.

(with respect to time) points of the three-dimensional space. The squared velocity in the logarithmic scale is called the energy. The energy plot is characterized by the short-term jumps which are rather well separated from two sides using the segmentation algorithm.

Let us approximate the signal time instants for all five participants of the experiment in two ways: using the logical convolution for *X, Y, Z*, and energy.

Table 3 gives the results of finding each of eight time instants for all five participants. Thus, the number –0.08 in column V for participant 2 and in row 47 means that for a time instant of the 47th second the best approximation for the segmentation based on energy makes 0.08 s, and the segment boundary is situated earlier than this signal time instant. The similar approximation equal to –1.62 obtained based on the logical convolution is given in column *F*.

Table 3 (right) gives the total revelation for each signal time instant. It can be seen that the culmination time instant (174th second) and the subsequent 199th second when participant 4 made a mistake were detected on the stabilographic level by all participants. The following time instants are given in the order of revelation: the 47th and 84th seconds when participant 2 first raised the buying price to 50 and then began selling at these prices to supporting partners.

Below the total degree of revelation for participants is given. The largest degree of revelation was demonstrated by participants 2 and 3. It is interesting that these participants have the maximal profits (0.62 and 1.16, respectively). Participant 4 has the minimal profit, and he possesses the lowest level of revelation of signal time instants. In essence, he reacted to a culmination time instant of the 174th second only and gave the erroneous decision at the 199th second. Probably the degree of revelation on the level of the functional state of signal time instants speaks of the adequacy of the process understanding by the participants, which contributes to receiving higher profit.

It can be seen from Table 3 that the author of the signal action in five cases out of eight was participant 2. In most cases, the negative values can be observed in Table 3, which means that the stabilographic jump precedes the decision making. The exception is the 174th second, but both methods (*V* and *F*) proved the high jump for participant 2 which lasted from the 167–168th to the 174th second, i.e., probably from the time instant of occurrence of the idea to decrease the selling price from 64 to 40 to the realization of this idea in the particular decision.

Negative numbers can be observed in rows for the 21th and 199th seconds (*F*) for participant 4. Making the erroneous decision was preceded by the high jump on the stabilogram of participant 4. Negative numbers in the row corresponding to the 140th second can be found in the column corresponding to the author of the signal action (participant 5).

Thus, we arrive at the following model of interaction of the functional state dynamics and the decision making process. The stabilogram registers the cogni-

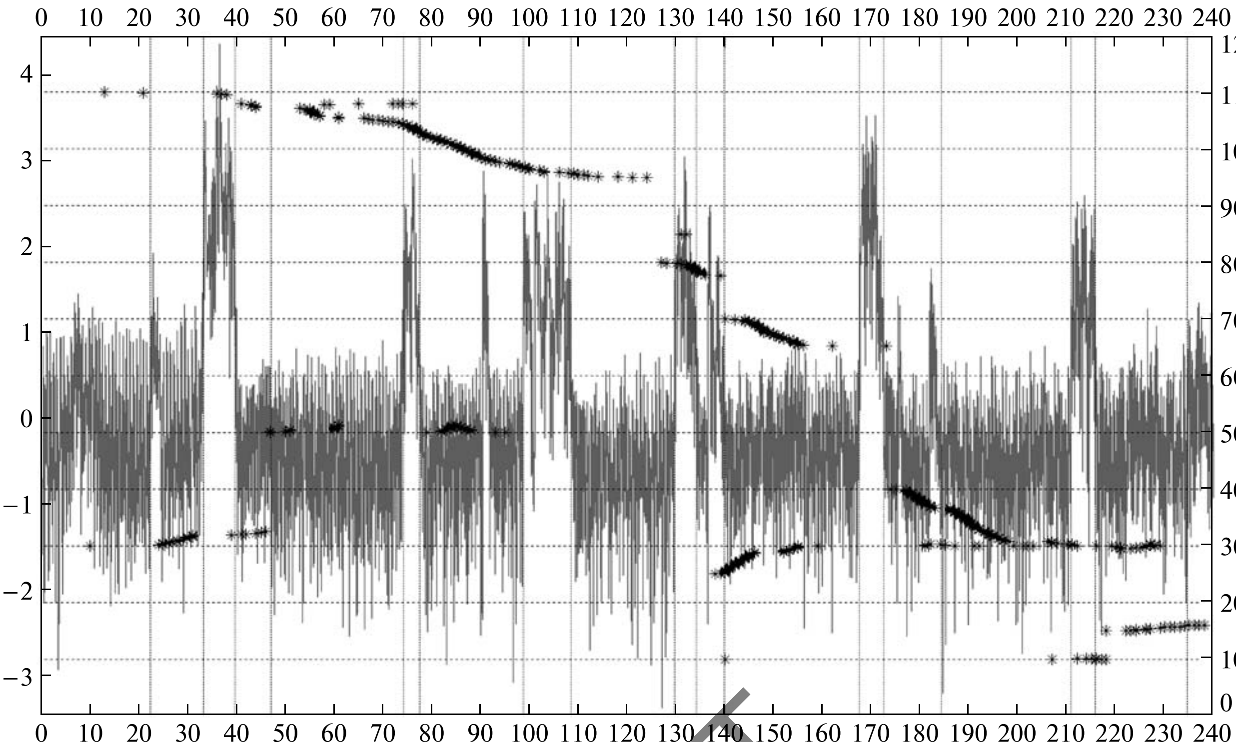

Fig. 6. Segmentation with respect to the total energy and request price.

tive load of the author of the signal action before the time instant of its implementation. The other participants react after the signal action is detected on the monitors. Of course, this is not a complete determinacy. For example, some participant might have decided to make the signal action, but was outstripped. One can speak of the tendency only. Another important conclusion is that the participant effectiveness depends on the degree of revelation of signal time instants. Participant 3 is of interest; he was not the author of any signal action, but he noticed all of them and successfully used corresponding information.

Since signal time instants are important for most participants, it can be assumed that the functional state dynamics expressed by individual stabilograms is not independent. Signal time instants given in Table 3 for most participants should serve for synchronization of the functional state of all participants of the group.

The simple method for detecting the synchronization of stabilograms is to calculate the total energy of the group and application the segmentation algorithm to it. Figure 5 shows the segmentation into nine parts of the group energy with the average energy values for each segment (logarithmic scale). Trading actions of the participants are indicated on this plot. Energy jumps show the time instants of the active beginning of the auction, the culmination time instant of revelation of information, and the final activity against the background of the rough mistake of one of the participants.

If the number of segments is doubled (Fig. 6), no considerable increase in the number of segments with important jumps of the total energy is observed.

Another method is based on the calculation of the canonical correlation of stabilograms of the participants. More precisely, one stabilographic time series is separated for each participant. Usually either the *Y*coordinate or the energy is taken. Then based on the logical convolution the joint segmentation is found. The obtained segmentation is filtered: only those time instants are left for which the considerable jump is observed for most (given fraction) of participants. For each of the remaining segments the average canonical correlation which measures the degree of synchronization of stabilograms of the group is calculated.

Figure 7 shows the estimate of the synchronization of stabilograms of participants with respect to the *Y*coordinate. From the initial 36 segments ten were filtered based on the condition that the jumps should be not less than 0.1 of the standard deviation for not less than three participants. It can be seen that the synchronization of stabilograms of the participants is most pronounced at the culmination of the auction. For the energy with 36 segments and filtering for three participants and the jump value not smaller than three standard deviations (energy varies stronger than *Y*) 16 segments were obtained.

Figure 8 shows the culmination most clearly. Some other signal time instants can also be noticed.

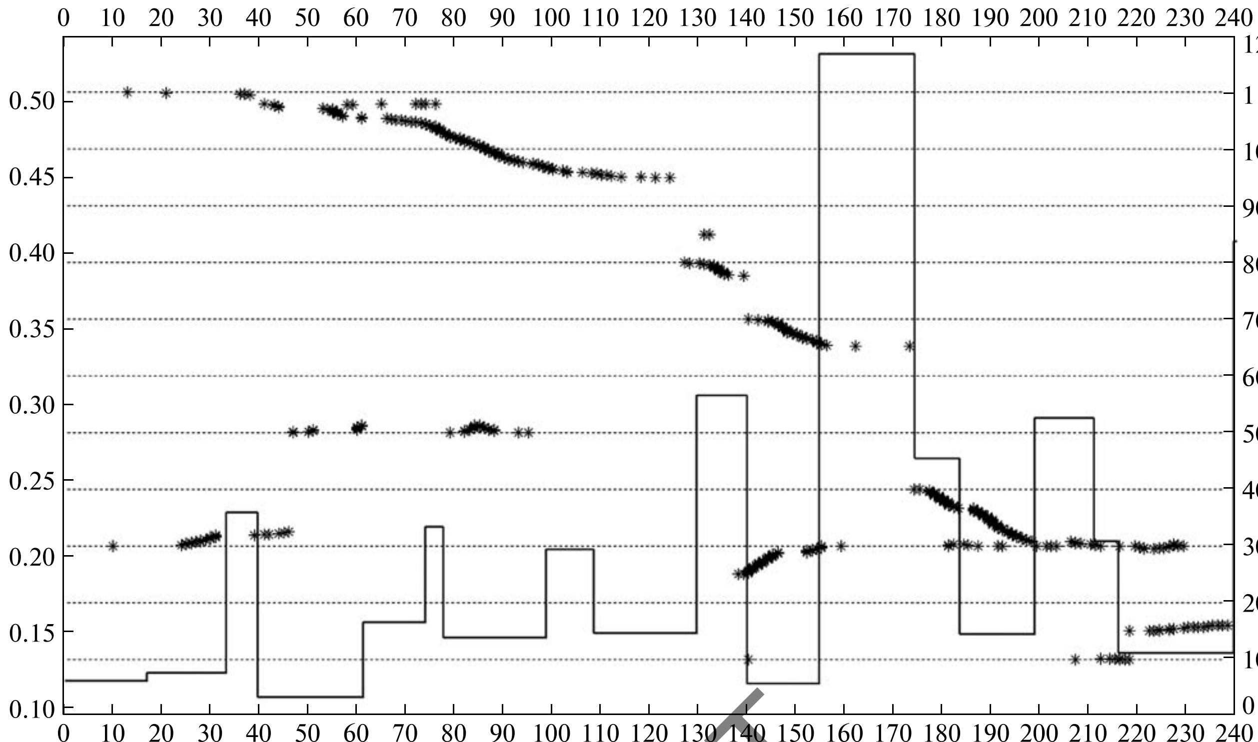

Fig. 7. Canonical correlation for filtered segmentation.

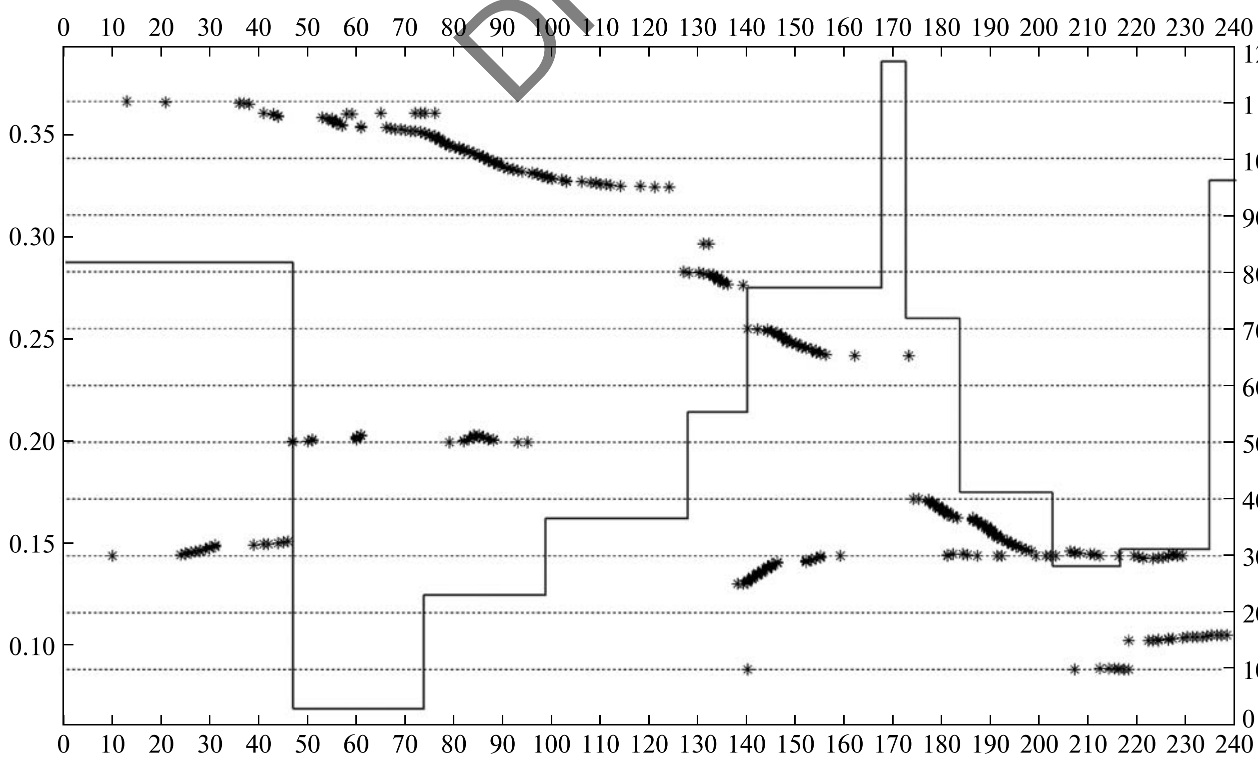

Fig. 8. Canonical correlation for energy segmentation.

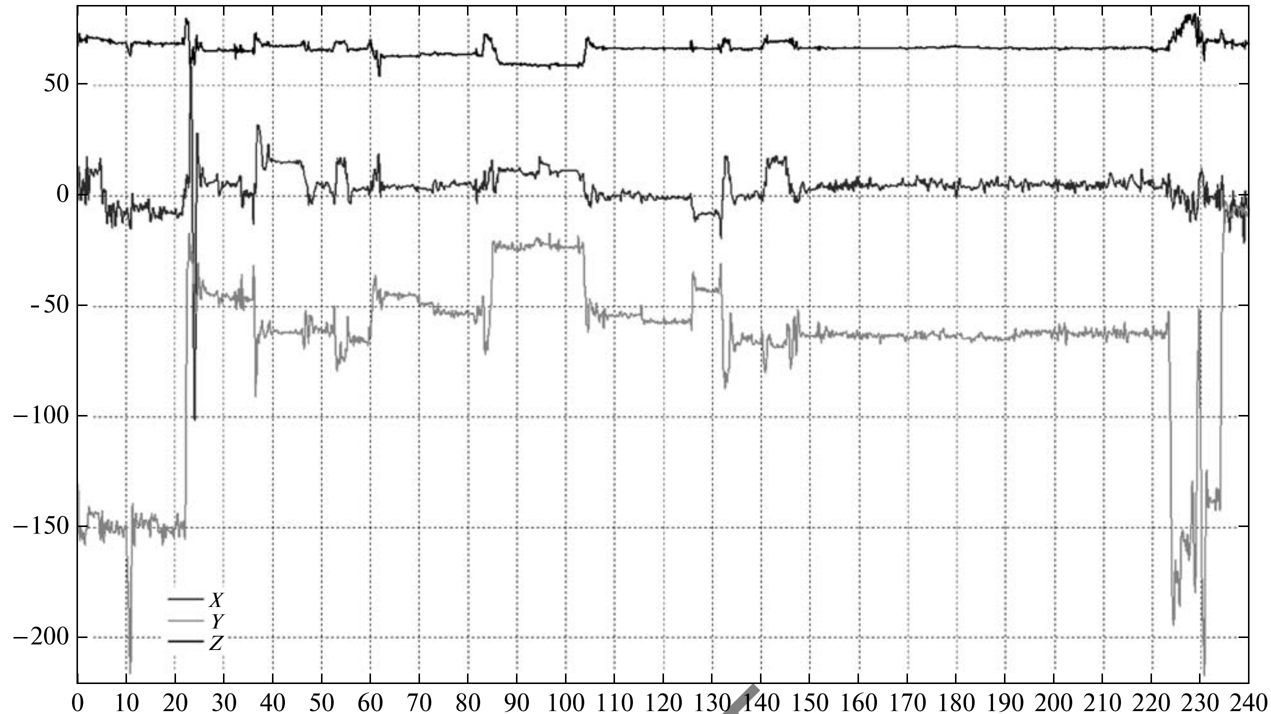

Fig. 9. Typical stabilogram of participant of reference experiment.

Thus, the following **model of interdependence of actions and functional state** of participants of the laboratory market can be formulated from the analysis of the results of this experiment. This model is based on the properties of experiment RE0 on information efficiency of markets; from the point of view of the game theory this experiment represents the dynamic game with incomplete information [2, 3]. In games of this class the player strategy determines his actions depending on the information available to him only. In this regard, the action of the player has the signal meaning. It is important to note that in our case signals are not free from charge, i.e., the trading actions of the participant determine his profit. This restrains him from attempts to confuse the market by false signals.

Not all actions of players in RE0 bear the similar signal load. The time instants of passing levels of defined values and sharp (above some threshold) variations of the request prices can be separated. Thus, the set of signal time instants A is formed. Then stabilograms for each participant are segmented, and the matrices R_V and R_F with a size of $|A| \times |N|$, where N is the number of participants are formed. The matrices correspond to the segmentation methods: with respect to energy and using the logical coordinate convolution. For the basic experiment both matrices are given in Table 3. They show the difference between the time instants of segmentation and signal time instants, and for each signal time instant the closest segmentation time instant is taken. Columns correspond to different participants. Empty cells point to the fact that in the given neighborhood of the signal time instant the certain participant does not possess the segmentation time instant.

The matrices R_V and R_F characterize the degree of revelation of signal time instants by all participants. The largest value should correspond to the culmination when all participants come to understand the true scenario. At the same time, the degree of revelation of signal time instants by a certain participant reflects the adequacy of his perception of the events taking place on the market.

The degree of synchronization of functional states of participants at the culmination time instant is measured using the total energy and the canonical correlation method.

This model of interaction of the decision-making processes and functional states of participants was used for an experiment in 2008 which will be called the reference experiment for brevity. The experiment RE0 with the same parameters as in the basic experiment including the variant of a true value of 30 was chosen. It is natural that the participants of the experiment were different and manifested other qualities. For example, the participants of the basic experiment in 2006 performed 10 times as much trading actions as participants of the reference experiment in 2008.

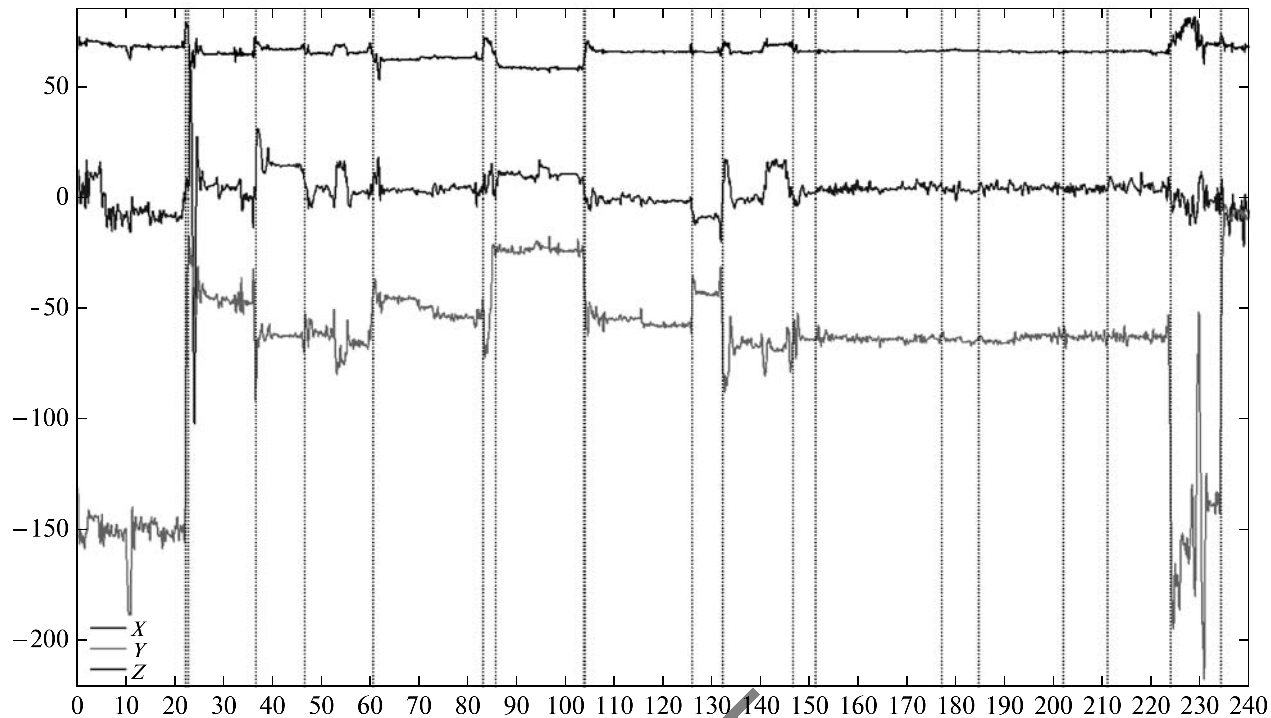

Fig. 10. Segmentation of stabilogram of participant of reference experiment.

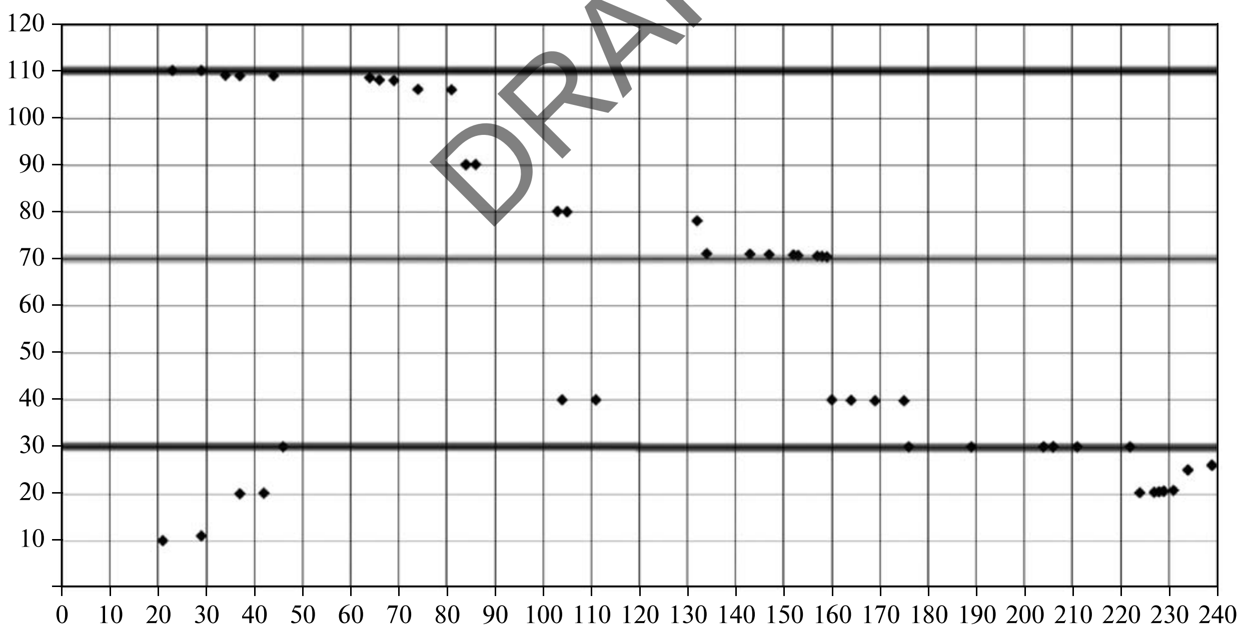

Fig. 11. Requested prices during the auction of reference experiment.

However, it turned out that the reference experiment also fits our interaction model.

Let us begin with the typical form of stabilogram of a participant shown in Fig. 9.

It can be seen that the property of local constancy of the average value preserves, which allows one to apply our segmentation algorithm (Fig. 10).

It can be seen that on the whole the segmentation is successful. Probably segmentation time instants near 180–210 seconds can be filtered, since here the jumps are much lower than at other time instants; however, one should not hurry to do it before considering the significance of this period in the auction history shown in Fig. 11.

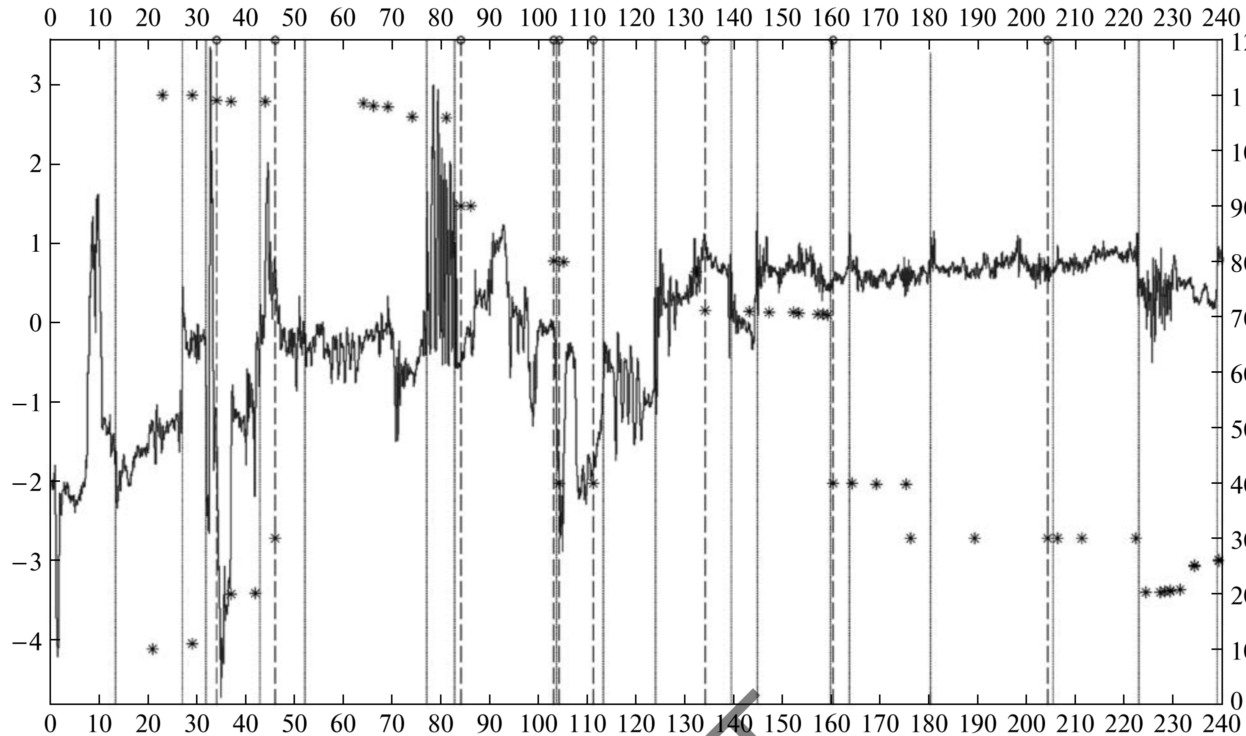

Fig. 12. Segmentation and signal time instants for one of the participants.

Similar to the previous consideration, the signal actions are:

(1) first (with respect to time) requests for buying at prices higher than 30, 70, and 110 (requests for prices higher than 110 are evident mistakes);

(2) first requests for selling at prices lower than 110, 70, and 30 (the latter request is the evident mistake);

(3) sharp change of the request price (larger than by some threshold value).

These actions correspond to the following signal time instants (in seconds):

34th—player 2 submits the request for selling at a price of 109 (information against 110);

Table 4. Revelation of signal time instants using segmentation

Participant		1		2		3		4		5		Revelation, %
Profit		2.30		3.22		2.40		3.20		3.28		
Method		V	F	V	F	V	F	V	F	V	F	
Time instant, s	Signal author											
34	2		3.42	−1.16	−3.92	2.06	−2.18	4.40	3.76	−1.64	2.70	90
46	2	−0.40	3.14			6.16	−3.24	−2.34	−1.60	−0.02	0.64	80
84	2	−2.94	2.62	5.3	2.34	−0.56	−1.38	−1.76		2.60	−0.80	90
103	2	3.70	4.2	1.62	0.24	0.18	0.44	−3.98	−0.10	−0.52	0.82	100
104	0	2.70	3.2	0.62	−0.76	−0.82	−0.56	2.98	−1.10	−1.52	0.10	100
111	1	4.98	3.24	4.58			2.02	−4.02		−2.76	−6.90	70
134	2	−6.34	5.12	−0.06	0.28	4.70	5.26	5.56	−2.12	0.06	−1.80	100
160	3	0.64		−0.08	−5.36	−1.42	−0.42	5.60	0.06	0.06		80
			−4.78	3.64	0.96	2.36	1.12	5.12	2.64	0.52	−2.06	90
	Revelation, %	77.8	88.90	88.90	77.8	88.9	100	100	77.8	100	88.9	

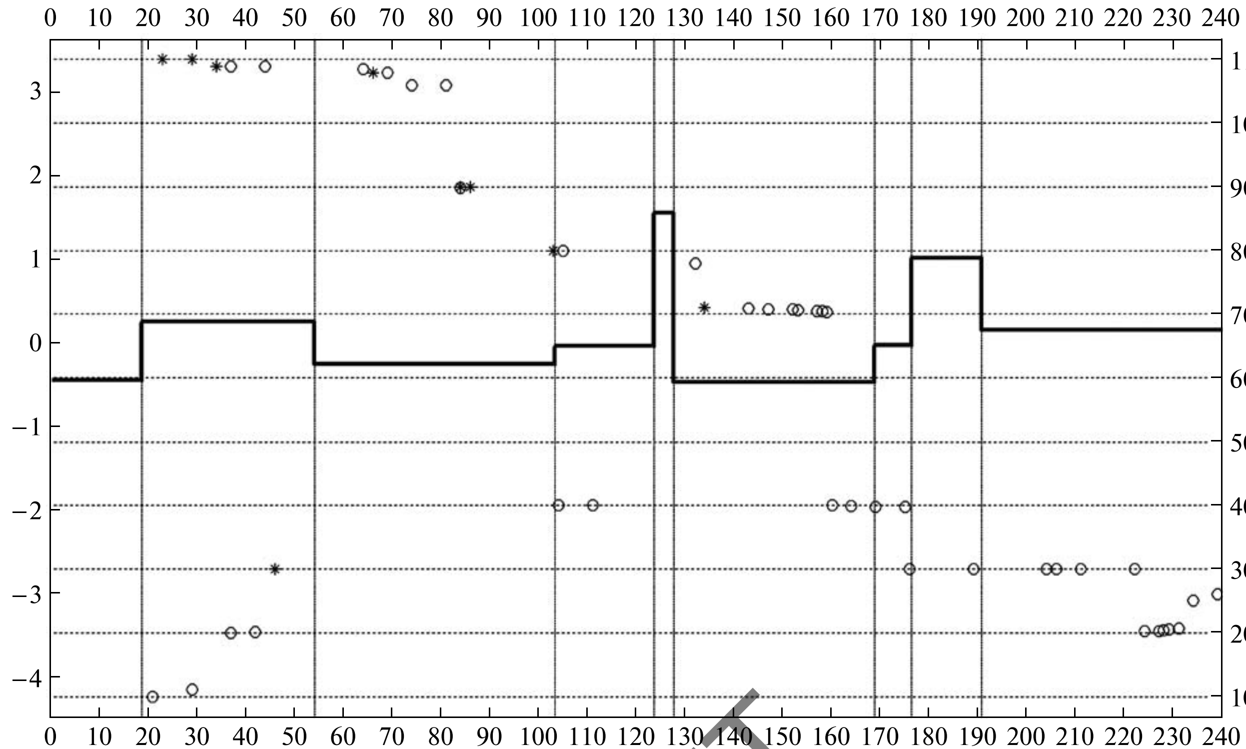

Fig. 13. Segmentation for total energy.

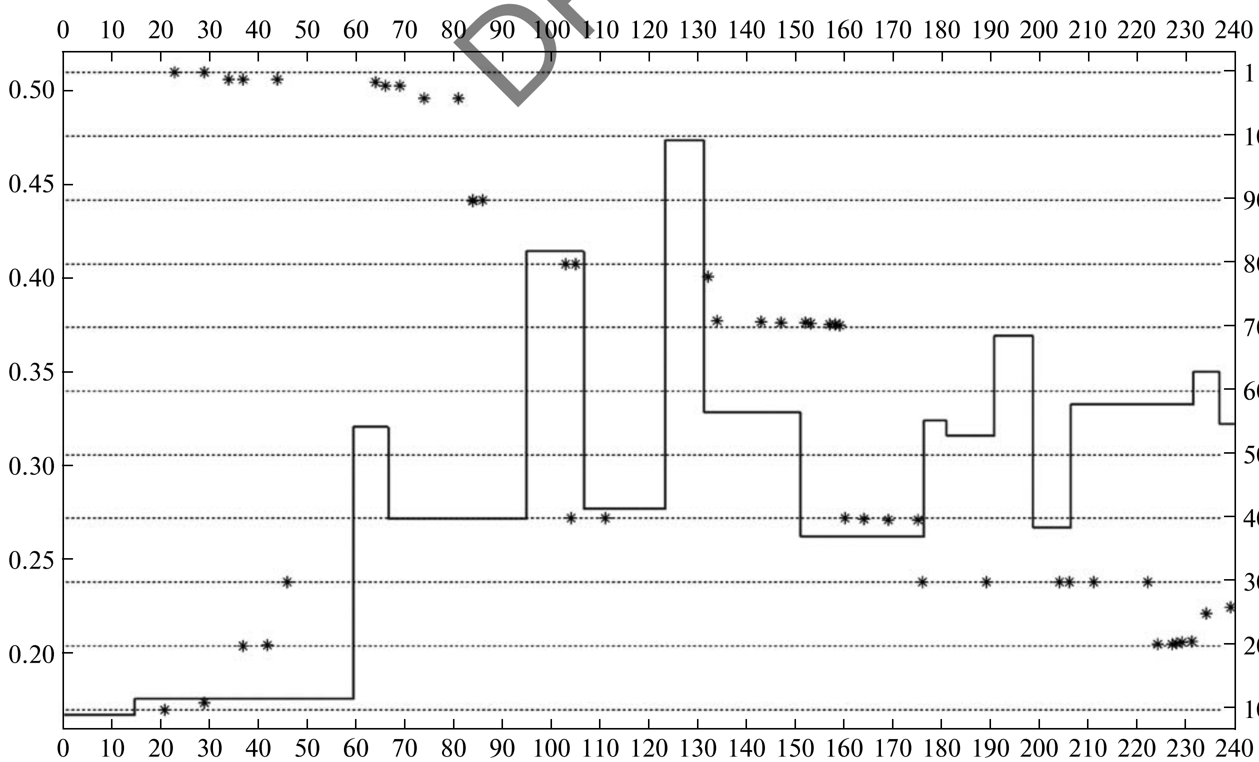

Fig. 14. Average canonical correlation for filtered segmentation.

46th—player 2 submits the request for buying at a price of 30 (the jump is observed);

84th—player 1 (possesses the information against a value of 70) changes the price of the buying request in a jump-like way from 105.9 to 89.9;

103th—player 2 reduces the selling price to 80 in a jump-like way;

104th—player 0 (possesses the information against a value of 110) submits the request for buying at a price of 40;

111th—player 1 sells to player 0 10 items at a price of 40;

134th—player 2 reduces the selling price to 71;

160th—player 3 submits a selling price of 40;

204th—player 3 performs the first deal at a price of 30.

Figure 12 shows the signal time instants and the segmentation time instants for one of the participants. It can be seen that for each signal time instant the close segmentation time instant can be found.

Table 4 gives the matrices R_V and R_F as one table. A much higher degree of revelation by participants of the reference experiment of signal time instants should be noted. Some participants (for example, participant 3) performed few trading actions, but their functional state was quite sensitive to the information received from the market at signal time instants.

The synchronization at the culmination at the 111–130th seconds is quite well manifested both for the total energy (Fig. 13) and especially for the average canonical correlation (Fig. 14).

Thus, the reference experiment proves the adequacy of the proposed model of the functional state of participants of the laboratory markets which represent the dynamic game with incomplete information with signal strategies.

ACKNOWLEDGMENTS

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