Calculation of the parameters of model hydraulic turbine using the value of the specific speed

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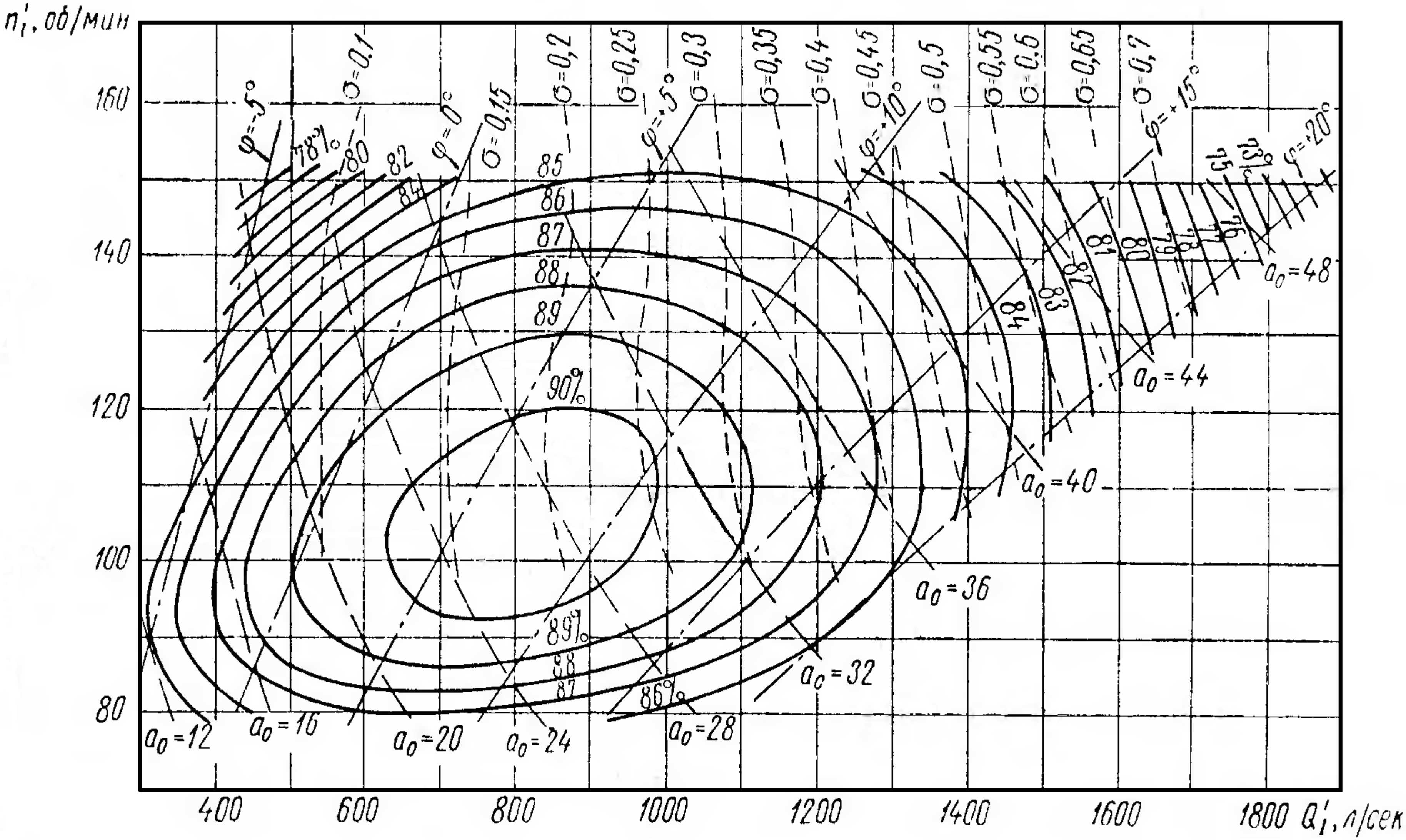
Yu.S.Volkov, V.L.Miroshnichenko, and A.E.Salienko.
Mathematical modeling of hill diagram for Kaplan turbine.
Journal of Machine Learning and Data Analysis. **2014**,
1(10):1439–1450

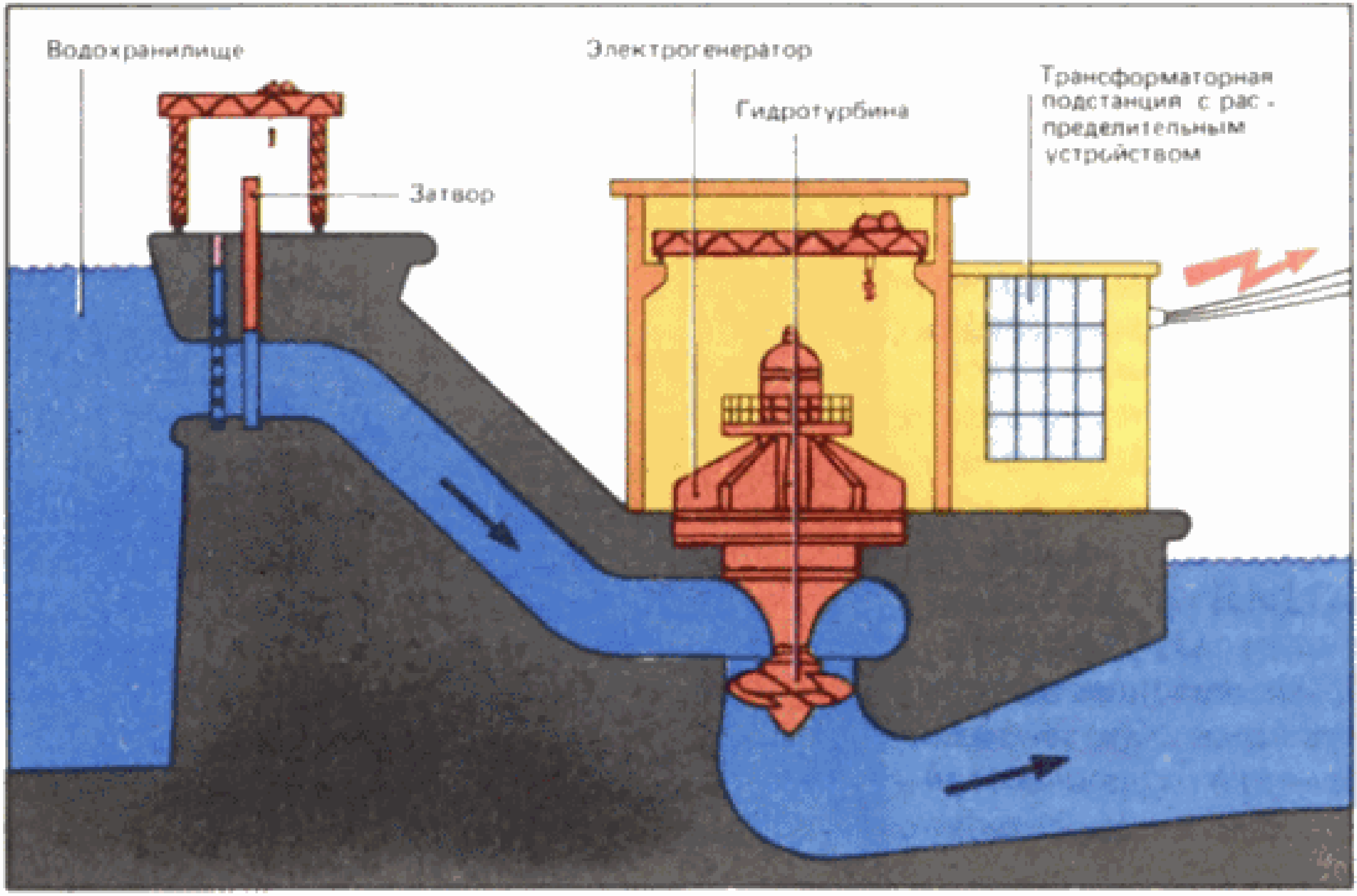
Picture 1. **Power test results of Francis turbine.**

These data are the values of efficiency function η of Francis turbine depending on frequency n (rpm) and discharge of water or flow Q (m^3/s)

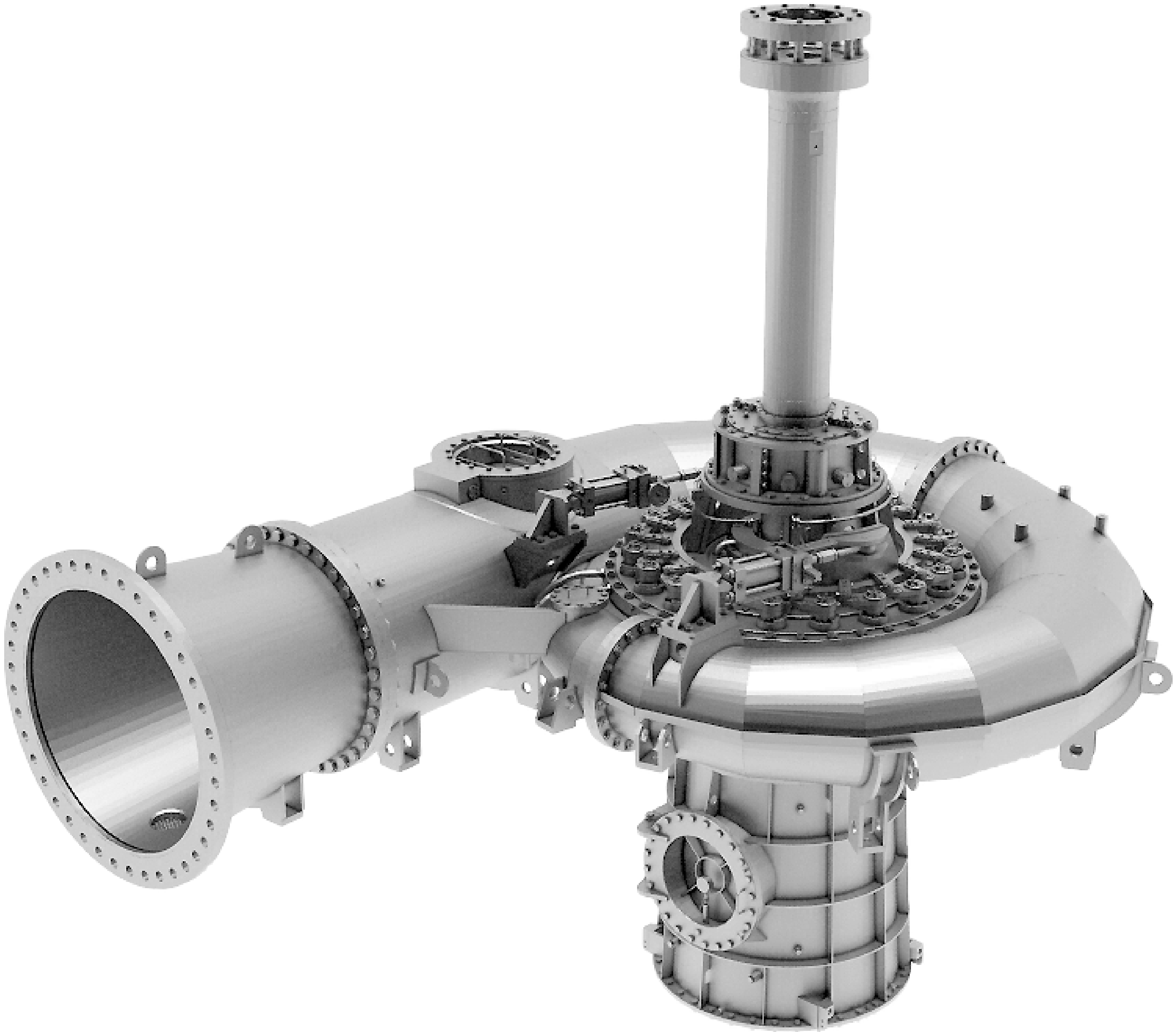
efficiency η is the ratio of the shaft power to the power flow $N = \rho g Q H$ (kW)
here H is the water head (m)



Picture 2. **Hill diagram.**



Picture 3. **Hydroelectric Dam.**



Picture 4. **The construction of hydraulic unit.**

Volkov Yu.S., Miroshnichenko V.L.,
Constructing a mathematical model of a universal characteristic for a radial-axial hydroturbine,
Sib. Zh. Ind. Mat., 1998, 1(1), 77–88.

D^m-splines

$$J(f) = \int_{R^2} \sum_{s=0}^{m} C_m^s |D^{s,m-s} f(x,y)|^2 dx\, dy$$

$$S = Arg \min J(f) \quad \text{where } f(P_i) = \eta_i \text{ for all } i$$

$$S_\rho = Arg \min\{J(f) + \frac{1}{\rho} \sum_i ||f(P_i) - \eta_i||^2\}$$

DMM-splines

Planning of hydroelectric plants.

The initial parameters for the planning:

head H (m), flow Q (m^3/s)

define the nominal power $N = \rho g Q H$ (кВт), i.e. the power of flow

The parameters to select:

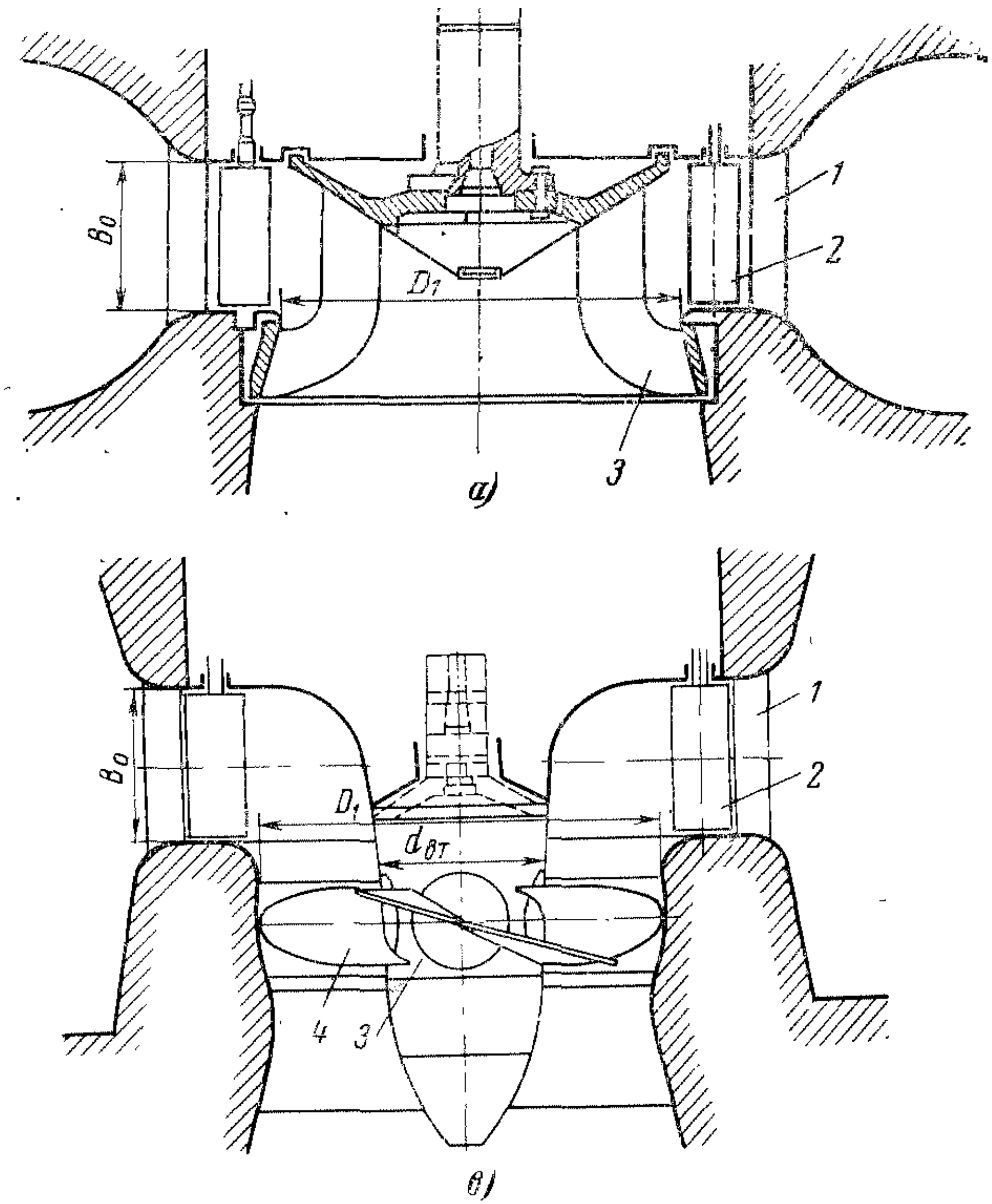
kind of hydroelectric unit

diameter of impeller D

frequency n

Three main systems of turbines

- [Francis] turbine that combines radial and axial flow
- [Kaplan] propeller-type turbine which has adjustable blades
- [Pelton] an impulse type turbine



Picture 5. **Francis and Kaplan turbines.**

A hydraulic turbine system may vary in size, construction of mechanisms, the configuration and relative dimensions of the flow path

Hydrodynamic qualities of working wheel defined by

- efficiency η
- cavitation coefficient σ
- specific speed coefficient n_s

η, σ, n_s as a functions depending on (D, n, Q, H) at all modes of turbine exploitation

Similarity Laws (scaling)

Reduced turbine parameters

$$Q'_I = \frac{Q}{D^2\sqrt{H}}, \qquad n'_I = \frac{nD}{\sqrt{H}}, \qquad N'_I = \frac{N}{D^2 H\sqrt{H}}$$

discharge, frequency and power of nominal reference-turbine with head $H = 1m$ and diameter $D = 1m$

The values of n'_I, Q'_I and N'_I in similar modes practically remain constant. In similar modes the dependences η, σ, n_s on main reduced parameters are almost the same.

Real data of model turbine testing

$a_0, Q'_I, n'_I, \eta, \sigma, \varphi$

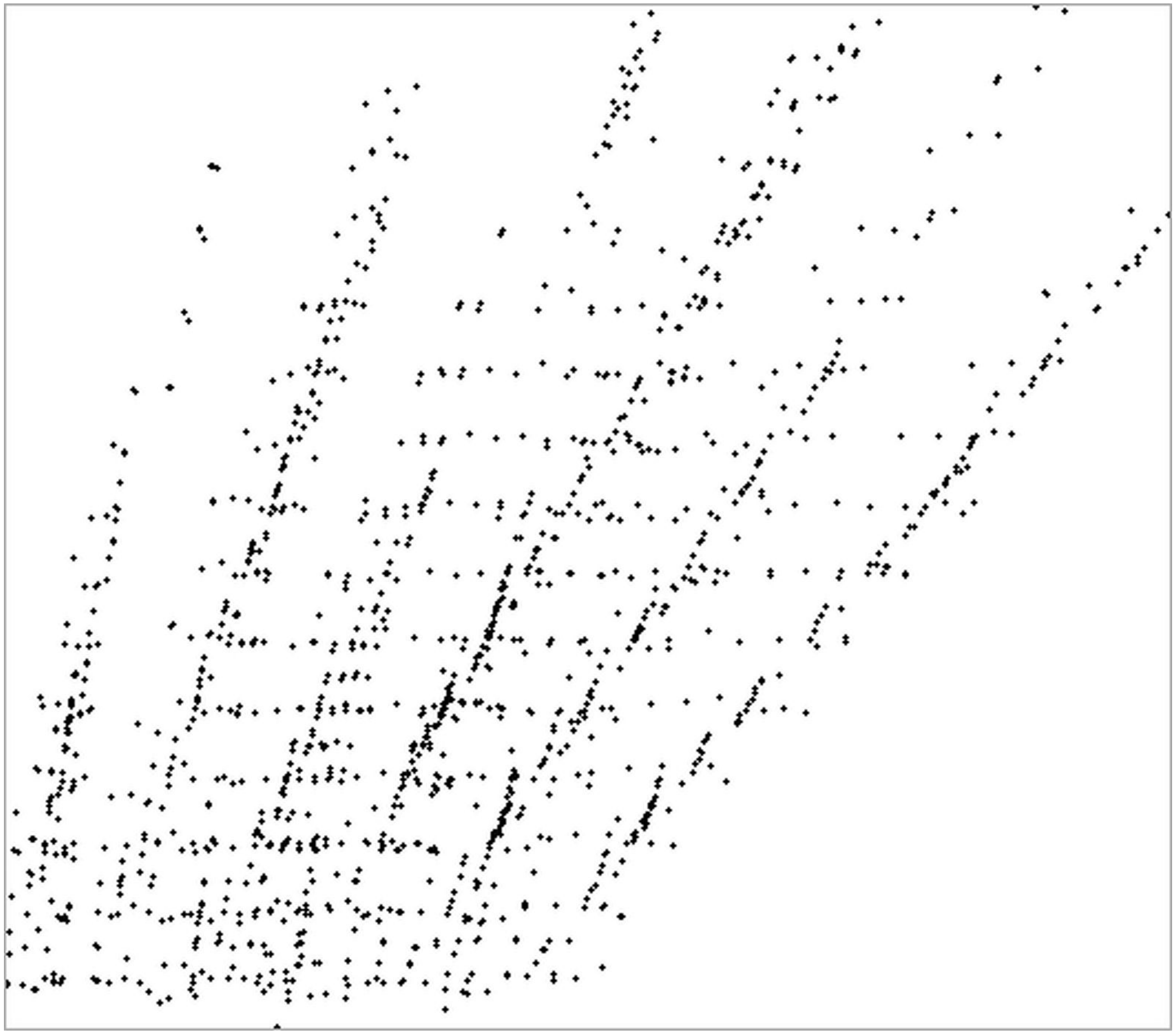
a_0 – value of opening of wicket gate

φ – rotation angle of blades

The problem is to recover function $\eta(Q'_I, n'_I, \varphi)$ and η as a function depending on Q'_I, n'_I, envelope of surfaces with parameter φ.

There are propeller and combinatorial characteristics, named universal characteristics

The example of real data of model turbine testing



Picture 6. **1479 points in coordinates** (Q'_I, n'_I) **for rotation angles of blades -10, -5, 0, 5, 10, 15.**

To construct a three-dimensional function on chaotic spaced data with high errors

DMM-spline

$$P(x, y, z): \;\; P_i(x_i, y_i, z_i) \in \Omega \subset R^3, \quad i = 1, \ldots, N,$$

$$f_i = f(P_i) \quad i = 1, \ldots, N.$$

DMM-spline of degree m:

$$S(P) = \sum_{i=1}^{N} \lambda_i r_i^m (\ln r_i)^{(1+(-1)^m)/2} + \pi_k(P),$$

$$\sum_{i=1}^{N} \lambda_i \pi_k(P_i) = 0, \quad \text{for all} \quad \pi_k \in \mathcal{P}_k,$$

where $r_i = r(P, P_i) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 + R^2}$,

R — Hardy's parameter,

k — polynomial degree of spline,

$\mathcal{P}_k$ — space of polynomials $\pi_k(x, y, z)$ — of degree k:

$$\pi_k(P) = \pi_k(x, y, z) = \sum_{0 \le i+j+l \le k} b_{ijl} x^i y^j z^l$$

Generalizations

- degree of additional polynomial is not associated with the degree of spline
- added the constant in formula distance

V.Bogdanov, W.Karsten, V.Miroshnichenko, Yu.Volkov.
Application of splines for determining the velocity characteristic of medium from a vertical seismic survey // Central European Journal of Mathematics. 2013, 11(4), 779-786.

Interpolating DMM-spline $S(P)$:

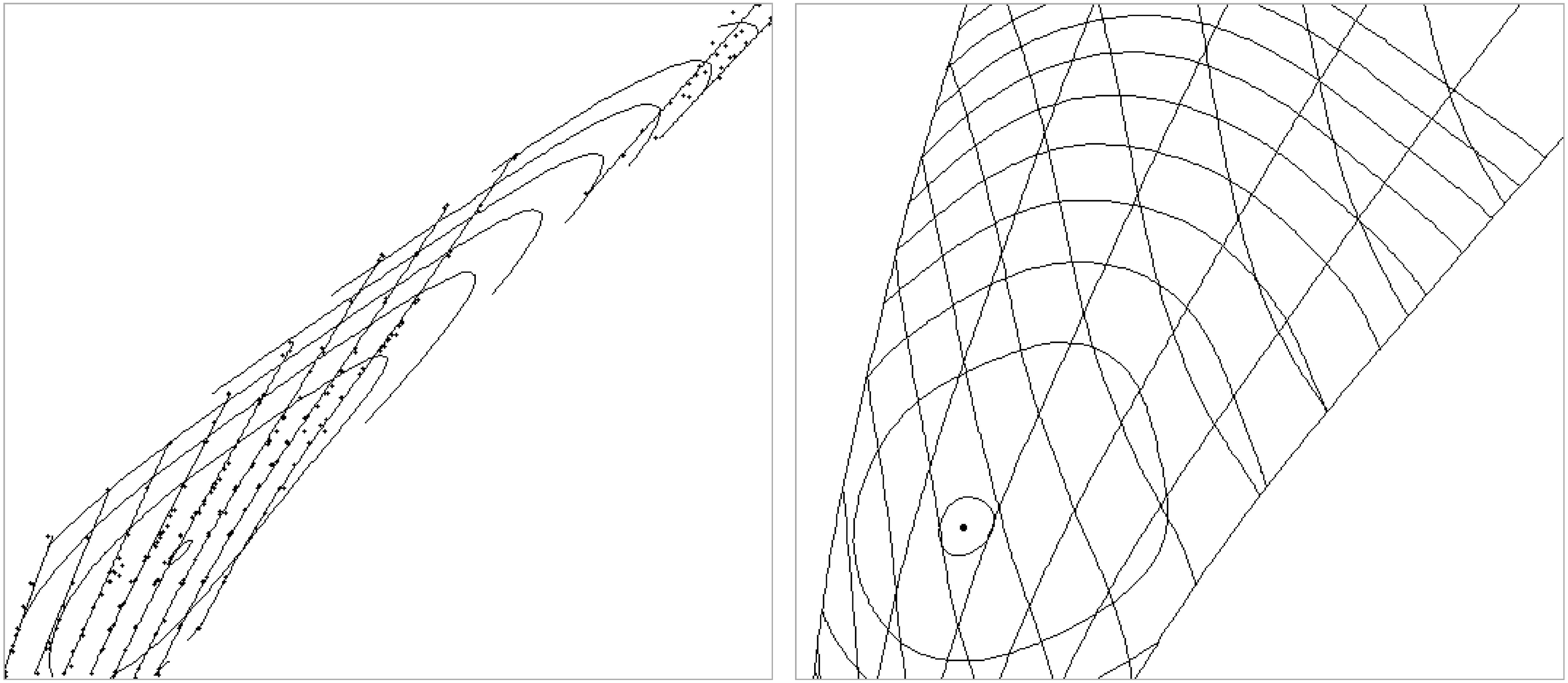
$$S(P_i) = f_i, \quad i = 1, \ldots, N.$$

Smoothing DMM-spline $S_\rho(P)$:

$$(-1)^{\tilde{m}} \rho \lambda_i + S_\rho(P_i) = f_i, \quad i = 1, \ldots, N,$$

$$\tilde{m} = [m/2] + 1,$$

$\rho > 0$ — smoothing parameter.

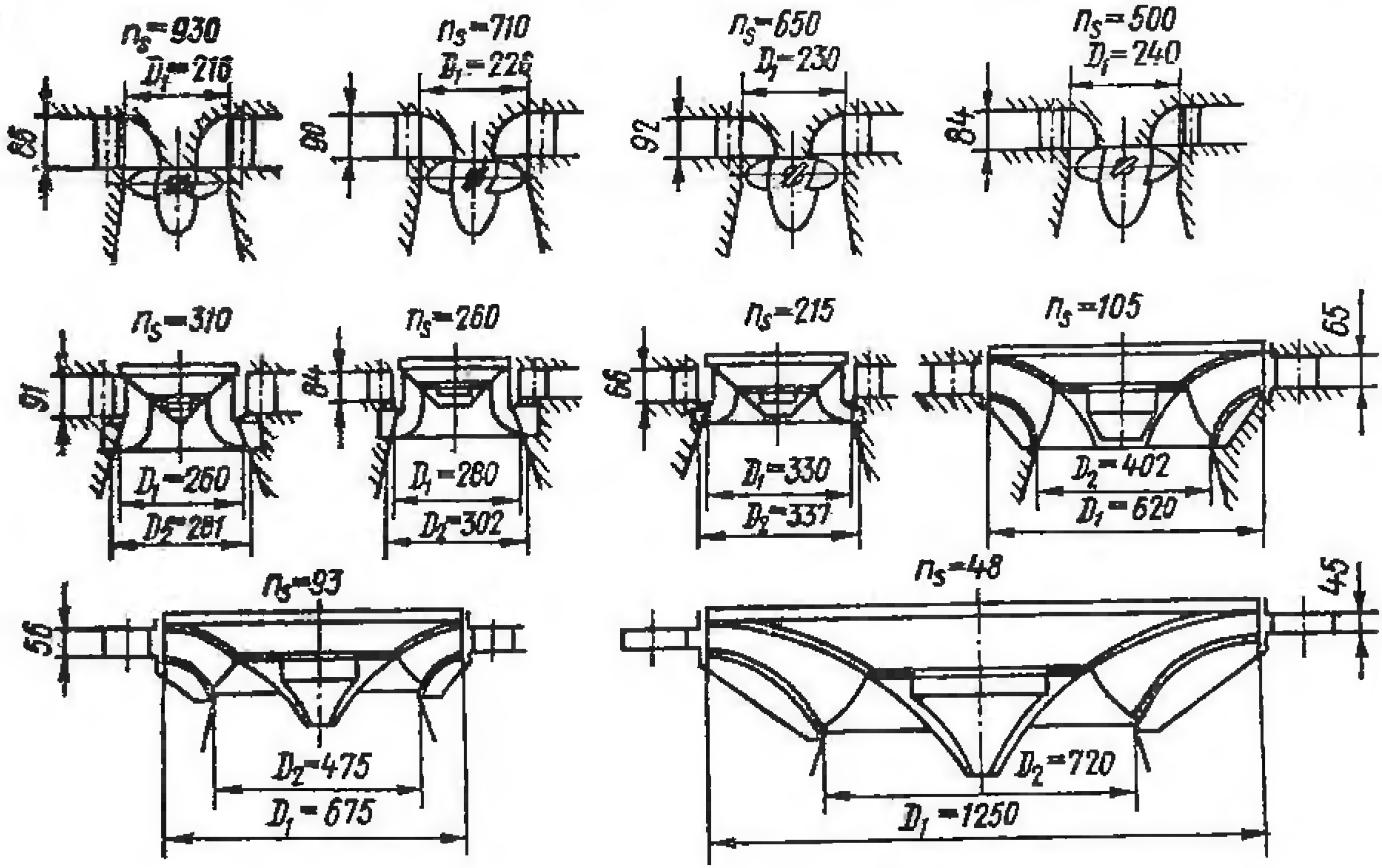


Picture 7. **Propeller ($\varphi = 0$) and universal characteristics.**

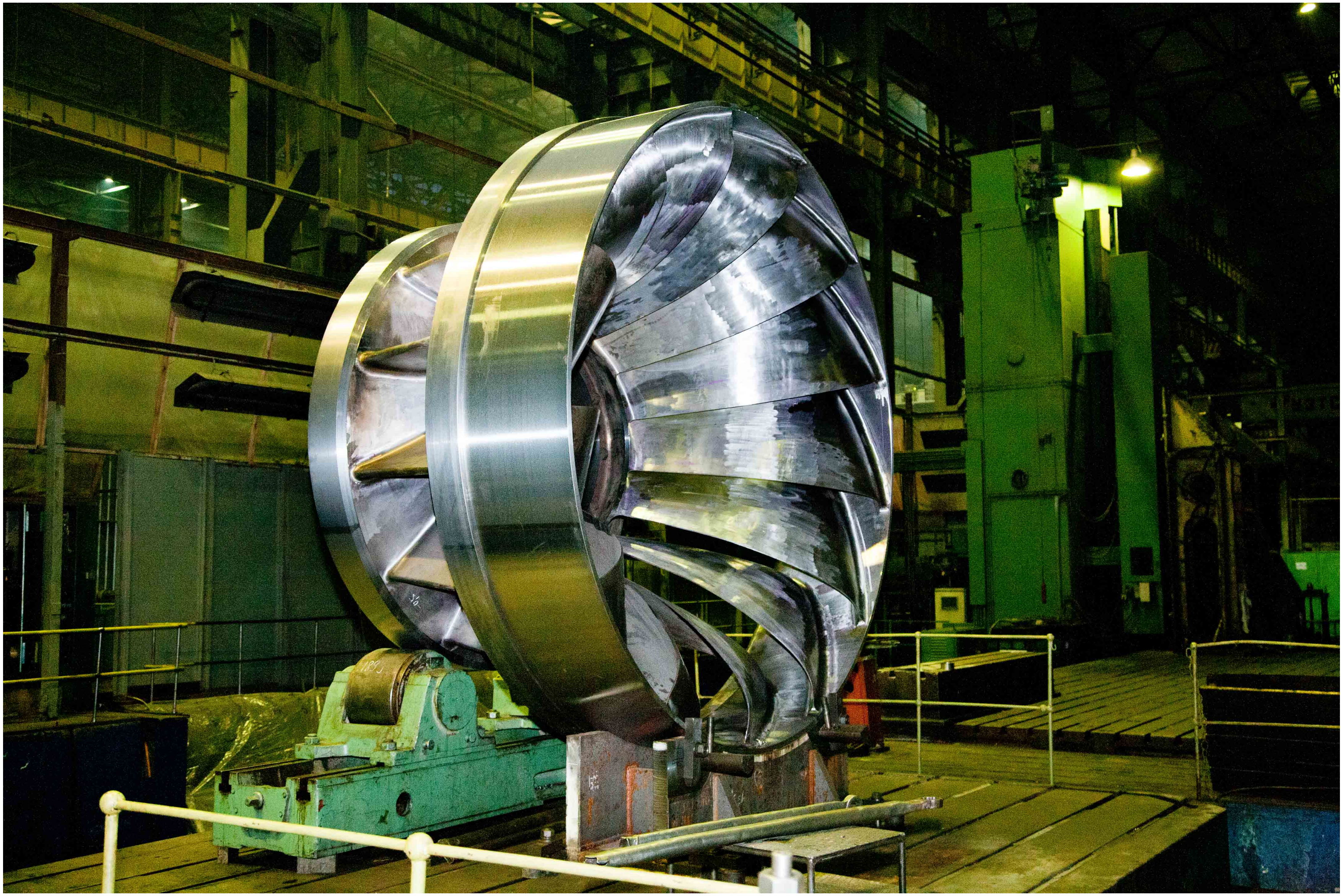
The specific speed

$$n_s = \frac{n\sqrt{N}}{H^{5/4}} = 3.65\, n'_I \sqrt{Q'_I \eta}$$

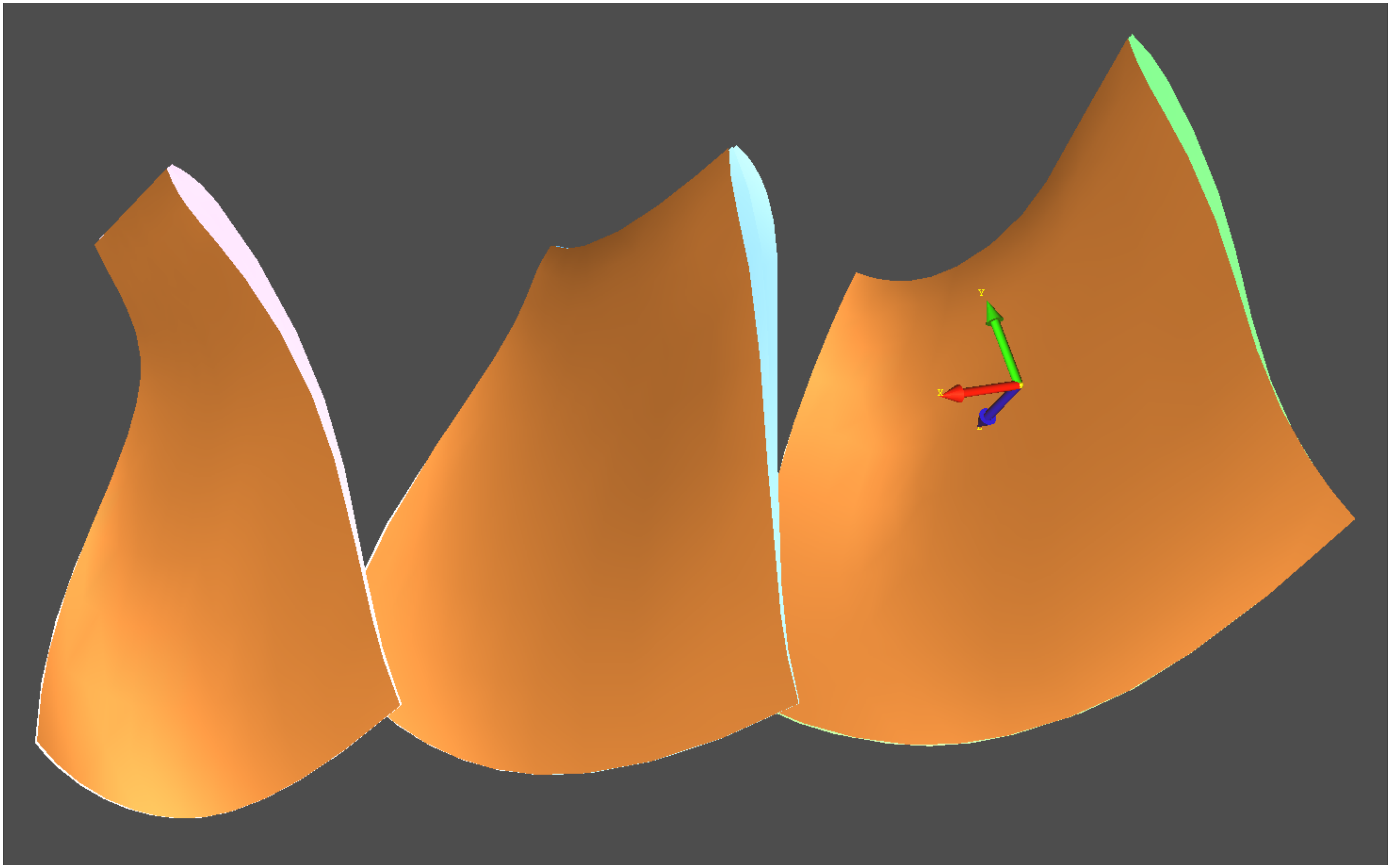
The specific speed of turbine is the rotational frequency of a turbine of a given type, but of a size that at a head $H = 1m$, turbine capacity is 1 h.p. $\quad (N_{h.p.} = 1.36\, N_{kW})$



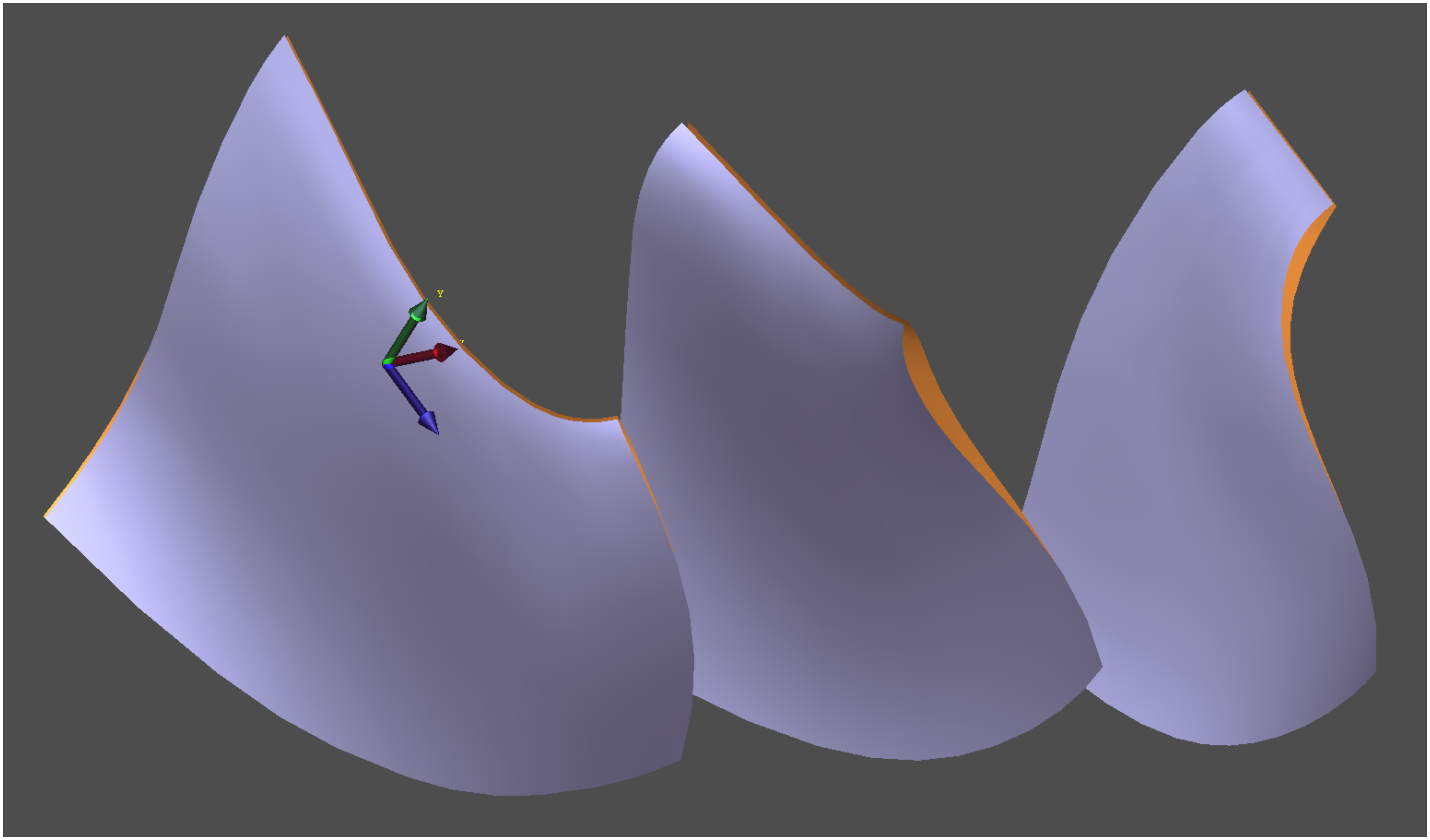
Picture 8. **The turbines with different n_s.**



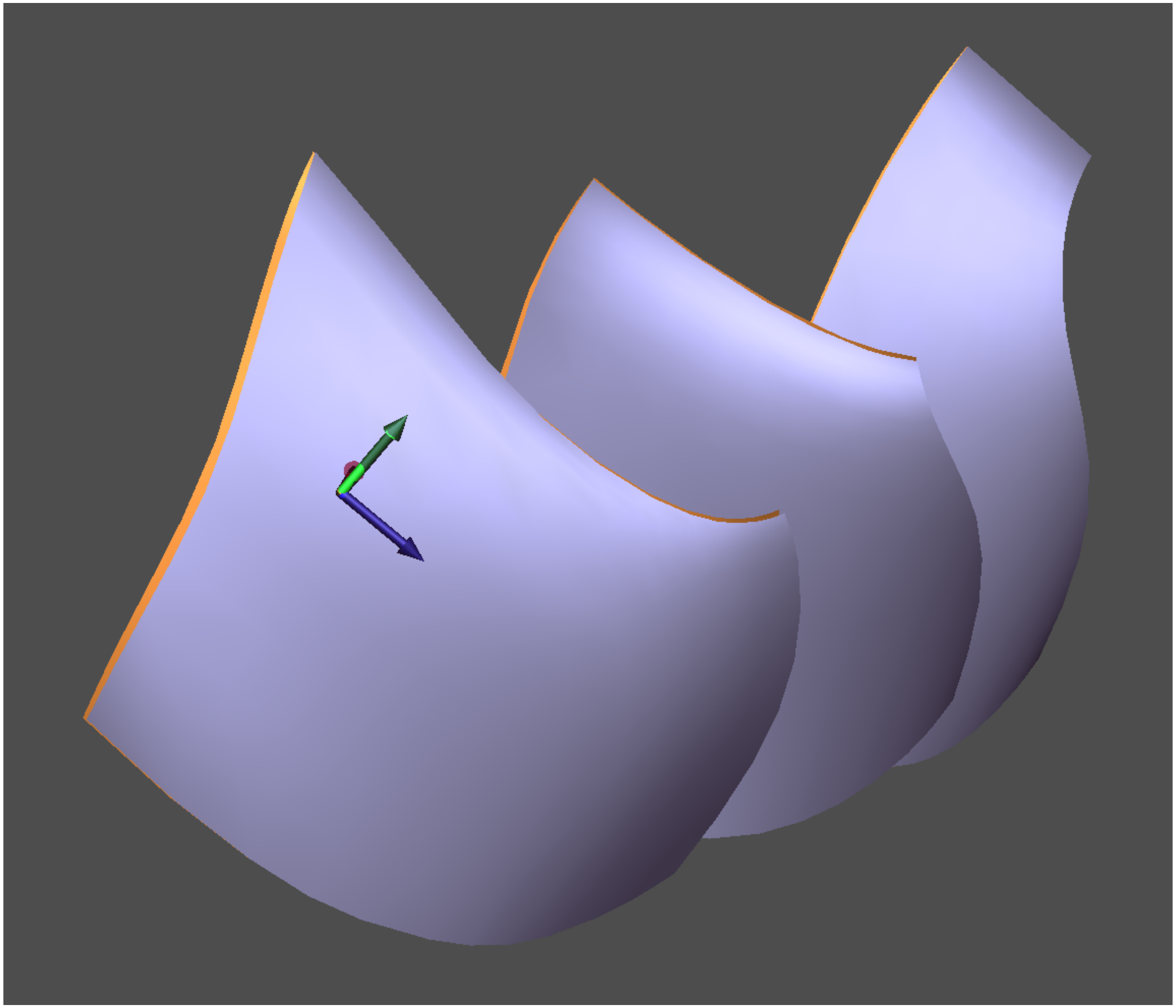
Picture 9. **Francis turbine.**



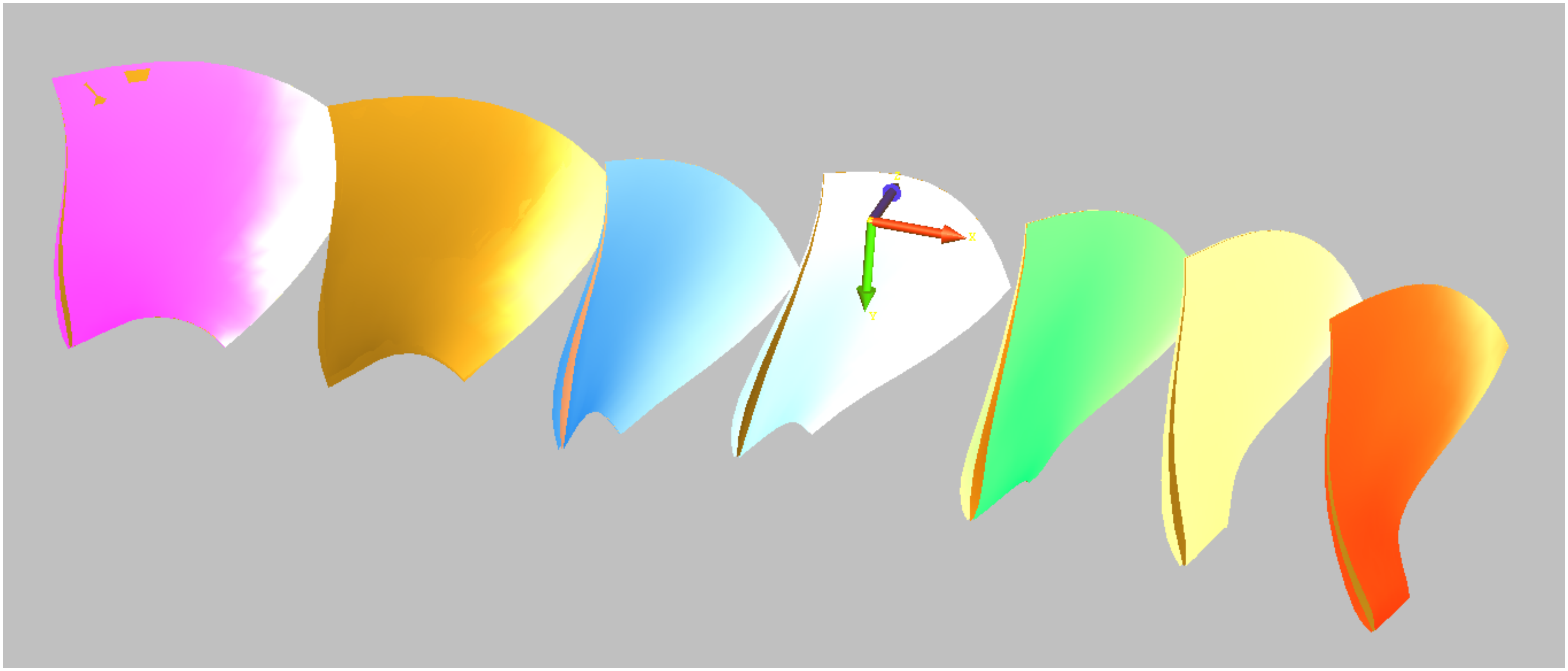
Picture 10. **Data models with $n_s = 100, 183, 317$.**



Picture 11. **Data models with $n_s = 100, 183, 317$.**



Picture 12. **Data models with $n_s = 100, 183, 317$.**



Picture 13. **Interpolation.**

Thank you

for attention!